

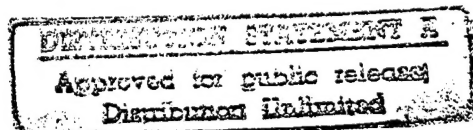
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# STRUCTURAL DESIGN CONCEPTS

SOME NASA CONTRIBUTIONS



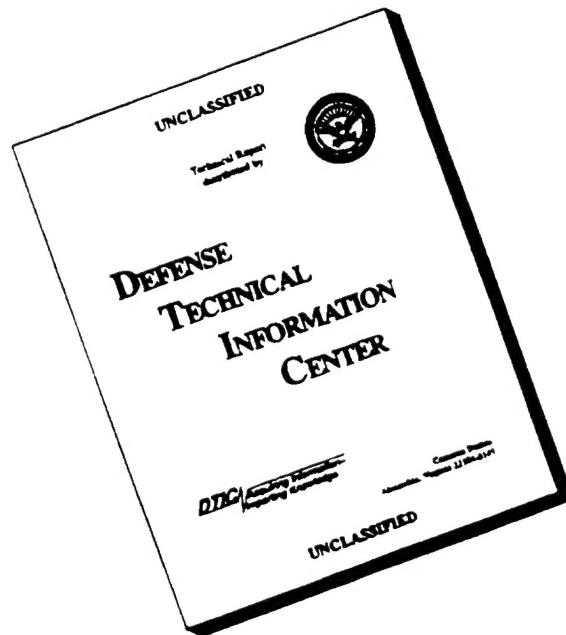
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# STRUCTURAL DESIGN CONCEPTS

SOME NASA CONTRIBUTIONS

By L. Albert Scipio  
University of Pittsburgh



*Technology Utilization Division*

OFFICE OF TECHNOLOGY UTILIZATION  
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
1967  
Washington, D.C.

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## Foreword

The National Aeronautics and Space Administration's work has accelerated progress in many techniques as essential to man's welfare on Earth as to the exploration of space. It has, for example, promoted the rational design of new composite materials for use in its structures. Simultaneously, its researchers and contractors have explored new structural concepts, and used electronic computers to help solve unprecedented design problems.

The Office of Technology Utilization strives to make the results of such work widely available. Prof. L. Albert Scipio of the University of Pittsburgh prepared this report on some of the structural design concepts with which NASA has been concerned. It is addressed to engineers and intended to facilitate their use of findings in the aerospace industry.

GEORGE J. HOWICK, *Director,*  
*Technology Utilization Division,*  
*National Aeronautics and Space Administration.*

## Acknowledgments

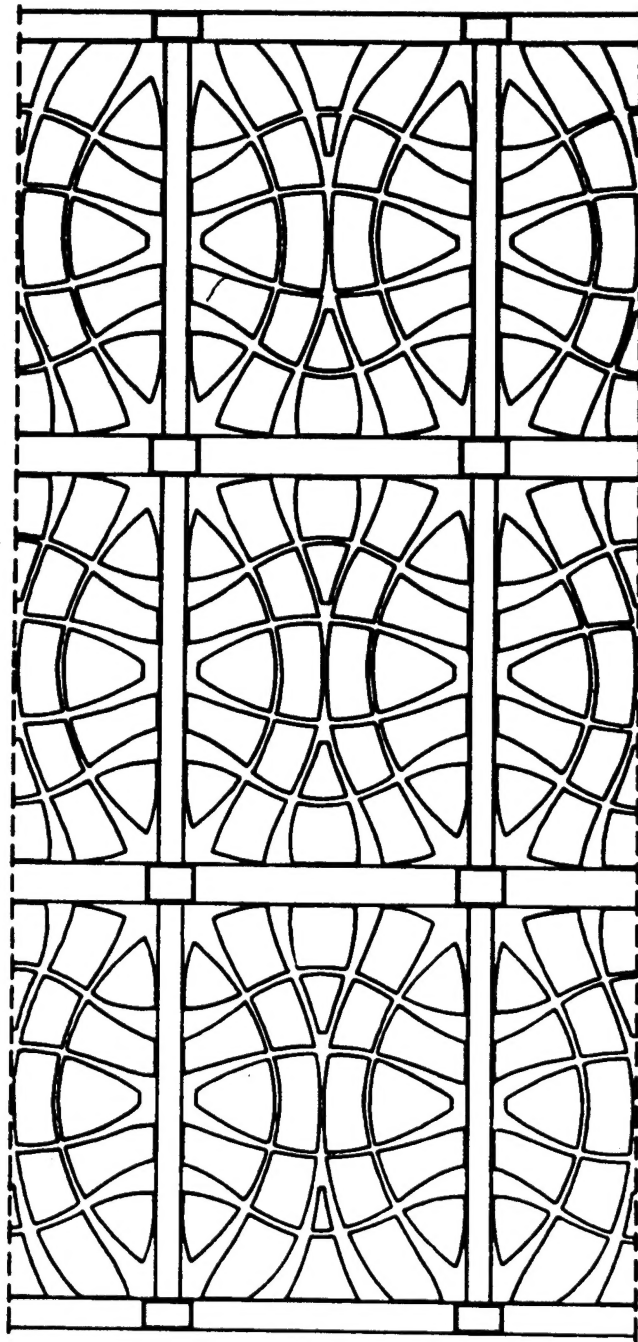
More persons and organizations than can be listed here helped the author prepare this survey. NASA personnel whose advice and assistance he would like to acknowledge include Dr. Richard L. Leshner, Assistant Administrator for Technology Utilization; George J. Howick, Director, Technology Utilization Division; and Technology Utilization Officers at the Goddard Space Flight Center, Lewis Research Center, Langley Research Center, and Marshall Space Flight Center. Specialists whose help was especially appreciated include Roger A. Anderson, Assistant Chief, Structures Research Division, Langley Research Center; J. F. Blumrich, Chief, Structural Engineering Branch, Marshall Space Flight Center; T. G. Butler, Head, Structural Analysis Computer Group, Structural Dynamics Branch, Goddard Space Flight Center; and R. H. Johns, Head, Structures Analysis Section, Lewis Research Center.

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An isostatic-ribbed plate can develop "grid action" in any direction. Nervi has used it in floor designs. (*Reprinted by permission of Prentice-Hall, Inc.*)

## Introduction

As man pushes back the frontiers of high-speed flight, changes in structural design requirements necessitate the development of new materials. During the last few years, material and structural engineers have solved many formidable problems in ways that have led to significant changes in the design process itself. They have been able to develop new materials and designs without sacrificing structural-weight to gross-operating-weight ratios. In fact, these ratios are generally comparable to, or in many cases better than, those for slower vehicles. This new knowledge and experience are providing a firm foundation for the design of structures to meet individual as well as space requirements.

This survey has three main aims: (1) to identify for those in the field of structural design the contributions of the National Aeronautics and Space Administration (NASA) and the programs it has sponsored; (2) to describe the development of construction materials associated with these advances; and (3) to suggest, by examples, some of the applications in which they may be used. The survey covers structural types (including material systems), structural concepts, and structural design synthesis and optimization. While the analyst may not find this approach as sophisticated as the designer will, he may gain from it some insight into the development of new structural design concepts. The degree to which these and other developments ultimately are utilized commercially depends on the foresight and ingenuity of structural designers.

Selection of materials and structural design to meet specific performance requirements is a complex problem. Each configuration and each part of the configuration must be examined and analyzed to provide the best possible structure for each application. Although we can only scratch the surface of the subject, we have attempted to offer some guidelines for structural designers in material selection, design approach, and optimization procedures for minimum-weight design.

## Structural Types

The basic structural element of the modern aircraft, missile, launch vehicle, and spacecraft is the thin-walled shell. Many structural types were proposed and tested to strengthen this shell wall; the more successful ones have become standard for similar types of structures. These include (1) stiffened skin, such as corrugation-skin construction; and (2) composite materials, such as fiber, particulate, flake, filled, and laminate composites. In this chapter, the basic characteristics of these structural types will be examined briefly.

### STIFFENED SKIN

One of the first steps in the structural design of a configuration is an investigation of the structural type that will best fulfill the strength requirements. The designer may have several possible choices. Here we will restrict our discussion to designs in which the use of heavy members is avoided. The designer then has the option of carrying the load pressure, bending, compression, and shear by means of semimonocoque structures. In our presentation, the term "semimonocoque" designates a skin structure that is stiffened by a number of reinforcing elements. Two general types may be used, the stiffened skin and the corrugation skin. Quite often both are referred to as "stiffened skin" or "reinforced skin" structures.

Stiffened-skin structures generally consist of reinforcing members that run in one direction only. These members can be attached with rivets, spot or fusion welded, or machined or "chem-milled" integrally with the skin. The bending and compressive loads are carried mainly by stiffeners, whereas the skin supports shear loads and twisting moments. A variation of this concept, which provides bidirectional rigidity, is the grid-stiffened skin. A system composed of stiffeners in two directions may be more efficient than one having either stiffener orientation alone.

The corrugation-skin structure is particularly efficient where the loading is predominantly unidirectional. The corrugation is assembled on the skin by riveting or open spot welding. A variation of this concept, which also provides bidirectional stiffening, is the waffle structure, in which a two-directional pattern is milled on one side of the skin.

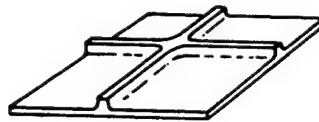
Figure 1 (from ref. 1) shows several versions of stiffened-skin construction. Reference 2 discusses the use of stiffened-skin construction in hypersonic vehicles.

Many theories of the semimonocoque have been developed, and the bibliography lists sources of a vast wealth of information. A recent paper by Hoff (ref. 3) deals with new advances in the analysis of semimonocoque structures. Recent developments in the analysis of orthotropic plates and shells are also fairly well documented in such sources.

Waffle -  $45^\circ$



Waffle -  $90^\circ$



Corrugation (1)



Corrugation (2)



Semi-Monocoque



Integral Stringer and Ring



FIGURE 1.—Types of stiffened skin construction.

## COMPOSITE MATERIALS

To meet requirements of high-speed aircraft, design engineers were forced to utilize materials up to, and beyond, their practical limits. The material barrier was a major obstacle to further development. To meet the demands, new material mixtures, or composites were developed; these were strong yet light and able to withstand severe temperature and corrosive conditions. Composites, in fact, seem now to represent "dream" materials with tailormade properties.

Composite materials are not new. The Babylonians are credited with discovering that chopped straw added to wet earth extended the life of a wall and enabled it to support more weight. The Egyptians used plaster of paris laced with hair to prepare their tomb walls. What these ancients discovered was that a mixture of materials is often stronger than any of the individual components. This concept is now well established in many technologies.

What do we mean now by a composite material? Definitions in the literature differ widely, as can be seen in references 4 and 5. The most appropriate definition of the composite materials covered in this book is: a mixture or combination of two or more macroconstituents that differ in form and/or material composition and that are essentially insoluble in one another. Although this working definition is not wholly adequate, it takes into account both the composition of the material constituents and the structural form.

A precise definition is difficult to formulate because of a scale factor (ref. 5). At the atomic level all elements are composites of electrons and nuclei; at the crystalline and molecular levels, materials are composites of different atoms; and at successively larger scales, materials may become new types of composites, or they may appear to be homogeneous (ref. 6). In our presentation, we will limit the discussion to the macroscale. Many metallic alloys that are composites of several quite different constituents become homogeneous materials on a macroscale. Specific examples of such materials are dispersion-hardened alloys and cermets. Some engineers will find our definition of composites too broad because it includes several engineering materials that are not usually considered composites, such as concrete, impregnated materials, filled plastics, and precoated materials. All such materials fall within the concept of composites, however, and should be treated as such.

Since our definition does not make a clear distinction between composites and composite structures, some combinations may be considered to be composite structures rather than composite materials. For example, there are differences of opinion as to whether a sandwich should be classified as a structure or a material. Although a precise



distinction is extremely difficult to make, the following discussion should be helpful in avoiding confusion.

Composite materials include mill composites: clad metals, honeycombs, nonmetallic laminates, and sandwiches produced in more or less standard lines and suitable for many different applications. On the other hand, we shall call "composite structures" those material systems that are designed and produced for a given application and that are also the finished structure, component, or product itself. Examples of composite structures are rocket nose cones (constructed of several integrated layers), tires (built up of several layers and a fabric-reinforced material), glass-reinforced plastic boats, and filament-wound vessels. Although a finished structure is also an integrated materials system, this does not preclude regarding it as a composite material. In general, structural engineers refer to all structures of complex or heterogeneous construction as composite structures.

The nature of any composite depends on the form and structural arrangement of constituents, which may include fibers, particles, flakes, laminae, and fillers. These structural constituents, shown in figure 2 (from ref. 5), determine the internal character of the composite. Since the structural constituent is generally embedded in a continuous matrix of another material, the matrix is called the "body" constituent. It generally encases the structural constituent, holds it in place, seals it from mechanical damage, protects it from environmental deterioration, and gives the composite form. Not all composites, however, have a matrix. Two or more different materials are sometimes bonded together, as in laminates and sandwiches. These layers form the complete composite.

Composite materials are divided into five basic groups by form of the structural constituents. (See fig. 3 from ref. 5.)

(1) Fiber (or fibrous) composites are composed of fibers in continuous or discrete filaments (called whiskers for their appearance in

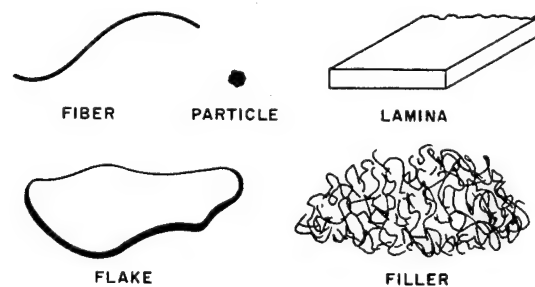


FIGURE 2.—Types of structural constituents. (Courtesy of Materials in Design Engineering.)

the production method) embedded in a continuous matrix. Fiber-fiber composites have no matrix.

(2) Particulate composites<sup>1</sup> are composed of minute particles, usually uniformly shaped, embedded in a continuous matrix.

(3) Flake composites are made up of flat particles or flakes, usually of isotropic material held together by an interface binder or embedded in a continuous matrix.

(4) Filled, or skeletal, composites have a continuous three-dimensional constituent which has a random network of open pores or passages, cells, or an ordered honeycomb, filled with another constituent.

(5) Laminar composites are formed by layers of single constituents bonded as superimposed layers.

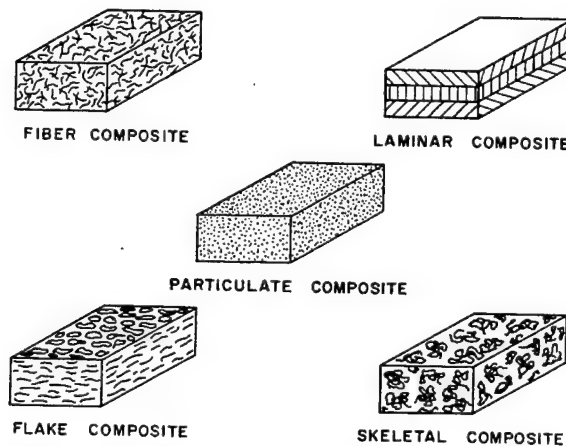


FIGURE 3.—Classification of structural constituents.  
(Courtesy of Materials in Design Engineering.)

### **Fiber Composites**

Fiber composites, particularly the fiber-matrix types, have been of interest to many structural engineers. The forms in which fibers can be employed in composites are numerous and draw on the long experience of textile technology for help and guidance (ref. 6). The simplest and most widely used arrangement is a mat of short fibers

<sup>1</sup> Flake and filled composites are sometimes included under particulate composites.

laid down in a random pattern. The material is essentially isotropic in its own plane; however, the strength and elastic modulus are determined by only a small proportion of the total number of fibers, oriented approximately in a certain direction. Highest strength in one direction is achieved when fibers in the form of continuous filaments are laid parallel to each other in a unidirectional pattern. This arrangement produces a high fiber-packed density. Fiber-matrix composites with unidirectional fibers are basically anisotropic. The highest strength is in the direction of the fibers, whereas strength in a transverse direction is essentially that of the matrix. Figure 4 summarizes the orientation, length, shape, and material characteristics of fiber constituents.

Fibers and matrices are available for a wide range of versatile

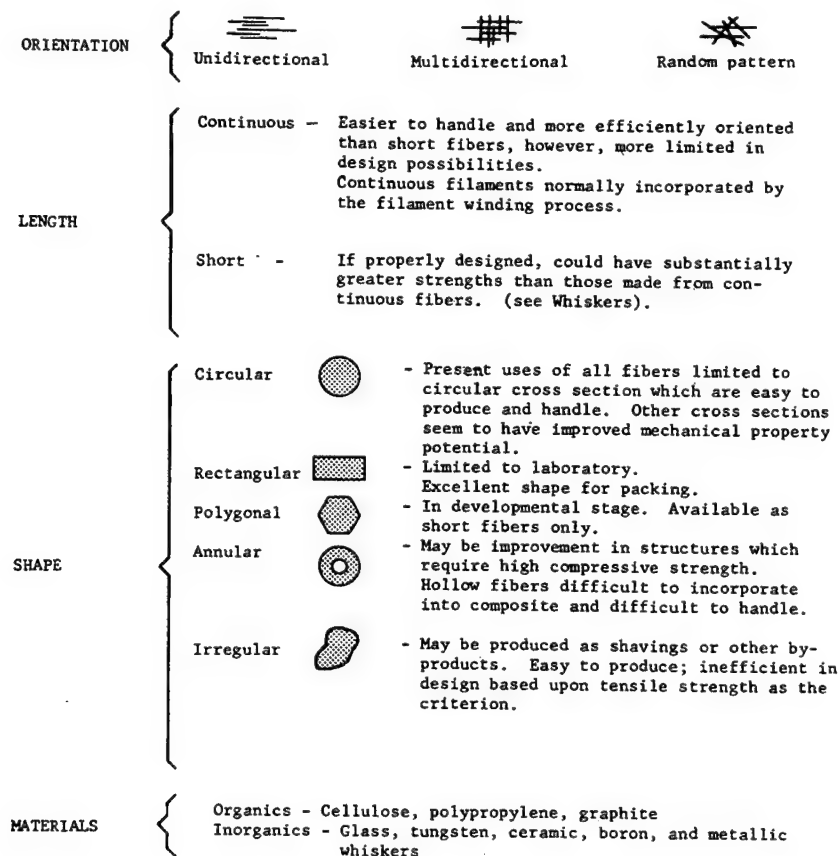


FIGURE 4.—Fibers in composites.

composites. Composites such as glass-fiber-reinforced plastics (GFRP), metal-fiber-reinforced plastics, and asbestos-fiber-reinforced plastics are fiber-synthetic resin combinations. Glass is the most widely used fiber, and synthetic resins such as epoxies, phenolics, and unsaturated polyesters are the most widely used matrices.

Development of fiber composites for high-temperature service has led to the use of high-temperature-resistant fibers in high-modulus metal matrices (refs. 7, 8, and 9). Alumina- and tungsten-fiber-reinforced silver; and carbon-, graphite-, and silica-fiber-reinforced aluminum are examples. Until recently most of the work was done with strong, stiff fibers of solid, circular cross sections in a much weaker, more flexible matrix (such as glass fibers in synthetic resins). At present, there is considerable interest in hollow, metal and ceramic fibers of noncircular cross sections embedded in stronger, stiffer, and more heat-resistant matrices. Although limited quantities of these new composites are available for high-performance applications, insufficient production and high cost restrict their use. More economical fiber composites, such as glass-fiber-reinforced plastics, are now coming into their own for structural applications where high strength and light weight are desirable.

Although the most commonly used fiber composites may be fabricated by various techniques, we shall limit our discussion to the use of continuous filaments for the fabrication of filament-wound structures and of whisker composites.

#### Filament-Wound Structures

The modern era in composites began with filament-wound plastics used in glass-reinforced structures such as pressure vessels. Extremely high strength-to-weight ratios are achieved. By exploiting the high strength of continuous fibers or filaments embedded in a matrix of a resinous material (either organic or inorganic), the winding technique is used to direct the structural strength. The resin contains the reinforcement, holds it in place, seals it from mechanical damage, and protects it from environmental deterioration. Rovings are drawn through a resin bath and are wound continuously onto a form, or mandrel, that corresponds in shape to the inner structure of the fabricated part. This winding technique permits orientation of structural strength to resist stress from an imposed load. A wide range of properties can be attained, depending on the filament and resin materials, winding patterns, and configuration of products. Suitable shapes of filament-wound structures include surfaces of revolution or combinations of surfaces that are flat or convex. (See fig. 5 from ref. 10.) Figure 6 (from ref. 11) shows various types of winding patterns.

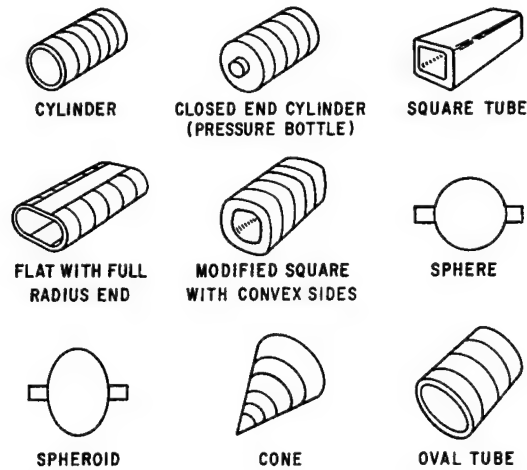


FIGURE 5.—Suitable section shapes for filament winding. (Courtesy of Machine Design.)

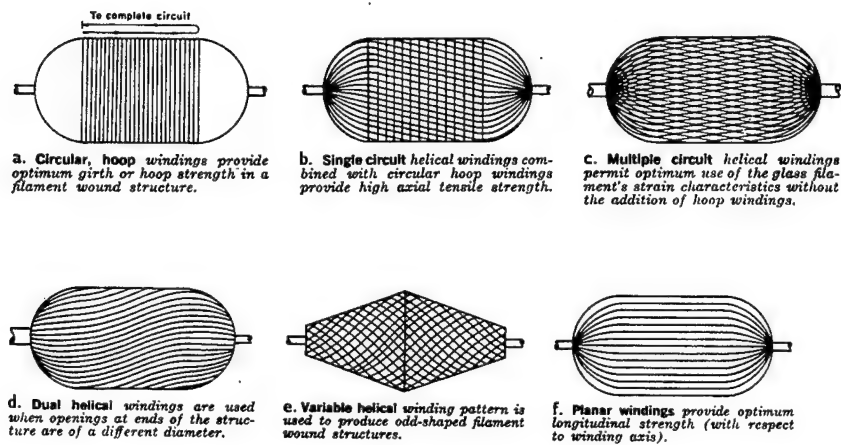


FIGURE 6.—Filament winding patterns. (Courtesy of Materials in Design Engineering.)

### Whisker Composites

One of the more promising composites under investigation is the whisker composite. Researchers have long been aware of the possibilities of developing materials that nearly approach their theoretical strength. Such materials have been found in the form of small fibers or filamentary microcrystals which are actually fine wisps of material

grown like mold cultures. Whiskers are extremely thin (about one-millionth of an inch to a few thousandths of an inch) and have nearly perfect crystal structures, with fewer defects than conventional materials. Consequently, some whiskers have tensile strength above 3 500 000 psi. Their extremely high strength-to-weight and stiffness-to-weight ratios make them particularly attractive reinforcements. Until commercial production is established, however, large quantities of whiskers will not be available for everyday products. One new technique for incorporating whiskers in a metallic matrix is to grow them directly in the metal (ref. 5) by the controlled unidirectional solidification of an alloy.

Among the future applications of whisker composites are whisker-reinforced plastics for bodies and basic structures in automobiles, whisker-reinforced metals for fully cast submarines, and whisker-reinforced concrete.

Analysis of a combination of materials that act in unison or interact is a complex task. The problem is complicated by the presence and interaction of many fibers, different stress levels in these fibers, differences in elastic moduli and Poisson's ratios between fibers and matrix, interaction of fibers and matrix, a boundary layer of indefinite thickness, and variable properties where the fibers and matrix interact (ref. 6). In addition, many matrices behave viscoelastically; that is, their behavior is time dependent. Therefore, a linear relation between stress and strain (Hooke's law) is not valid. Stress concentrations that develop around discontinuities and stress conditions occurring at the ends of fibers and at breaks in fibers also contribute to the difficulties of understanding the behavior of a fiber composite under stress.

For design purposes, a good approximation of fiber-composite properties is often obtained by application of a simple rule that says: the properties of the composite are the sum of properties of the individual components multiplied by their fraction in the total volume. Unfortunately this rule breaks down when the properties are complex functions of fiber geometry, spacing, relative volumes, etc. The determination of properties at right angles to the fiber direction is sometimes based on the weakest link hypothesis. This link is usually the matrix.

Detailed analyses of composites have been given in recent works by Tsai et al. (ref. 12) and Alexander et al. (ref. 13).

Fiber composites are now found in a wide variety of products including nose-cone shields for spacecraft, rocket motor cases, helicopter rotor blades, high-pressure tanks for liquid gases, storage tanks, railway tank cars, automobile fenders, automobile heaters, valves, ball bearings, truck bodies, walls of experimental homes, and concrete forming pans. Table 1 (from ref. 14) lists some present and future uses of fiber composites as primary structural materials.

TABLE 1.—*Present Development Applications of Whisker Plastic Composites*

Type of composites	Application	Nonproprietary information
Unidirectional: Reinforced plastic wire.	Bulletproof vest, deep-submergence cable.	Proven alinement, 19 million modulus, yarn formation.
Bidirectional: Reinforced plastic laminate.	Space helmets, turbine blades, high-speed centrifuges.	10 million modulus.
Tridirectional: Reinforced casting resin.	Dental research <sup>a</sup>	2× better than any other reinforcement.
Supplementary reinforcement— Transfer molding compounds.	Miniature rockets, electronic micromolding.	1.5 v/o addition=20 percent increase in tensile burst strength.
Interstitial reinforced Fiberglas.	Filament-wound deep submergence vessels.	0.65 v/o addition=38 percent increase in interlaminar shear.

<sup>a</sup> Research on reinforcing dental plastic, metal, and ceramic fillings.

### **Particulate Composites**

Particulate composites differ from the fiber and flake types in that the distribution of the particulate constituent is generally random rather than controlled. In some of these composites, the particulate constituent becomes dimensional only on the microscopic scale. These discrete particles are contiguous but insoluble and chemically unreactive with the matrix. The particulate constituent generally contributes strongly to the properties of the composite.

Particulate composites may be divided into several classes: (1) metal in metal, (2) metal in plastic, (3) metal in ceramic (including cermets), (4) organic in organic, (5) nonmetallic in nonmetallic, (6) dispersion-hardened alloys, and (7) self-lubricating alloys. Each of these composite classes is an individual subject beyond the scope of the present treatment, but examples may be found in reference 5. Only three specific types of particulate composites will be discussed here. These are (1) cermets, (2) dispersion-hardened alloys, and (3) self-lubricating alloys.

A cermet is a composite in which ceramic grains are held in a metal matrix in amounts up to 30 percent of the total volume. Cermets are among the most important composites and have a range of proper-

ties dependent on the composition and relative volumes of the ceramic and metal constituents. Of the various possible combinations, carbide-based and oxide-based composites are among the most widely used. Examples of carbide-based cermets include tungsten carbide, chromium carbide, and titanium carbide; examples of oxide-based cermets include aluminum oxide and magnesium oxide in chromium.

Dispersion-hardened alloy composites consist of hard, submicron-sized particles dispersed in a softer metal matrix; the particles are usually less than 3 percent by volume. These composites differ from cermets in the smaller size of the particles and by their lower proportion of concentration. Although both the size and the proportion of the total volume of the dispersed particles are small, these particles control the strength properties of the composite. The finer the particle size and the smaller the spacing, the better, generally, the properties (ref. 6).

Self-lubricating alloys are a recent development based on the dispersion of dry lubricant powders in a metallic matrix. These composites include combinations such as molybdenum disulfide or tungsten disulfide in nickel, boron nitride or calcium fluoride in steel, and tungsten diselenide in copper or silver.

Concrete is perhaps the oldest particulate composite. Most recent advances in particulate composites, however, have been related principally to the aircraft industry and the nuclear field. Aircraft builders have used sintered aluminum powder alloys (SAP) for impellers and pistons, and in the nuclear field, cermets are used for applications such as valve seats and bearings. Other structural uses of cermets include high-speed cutting tools, integral turbine wheels, and nozzles. Structural uses of steel-particle-filled plastics include small-lot production tooling. More exacting industrial demands for heat- and oxidation-resistant materials should lead to increased use of metal-in-ceramic particulate composites and the use of metals and nonmetallics in plastics.

By treating the particles as inclusions and applying the theory of stress concentrations (around discontinuities of various shapes), it may be theoretically possible to analyze the resulting structure. Some work has been done and solutions have been obtained for a circular particle embedded in an infinite matrix. As greater numbers of inclusions are considered, this approach becomes impractical, however, and composites consisting of small particles randomly distributed are generally treated as conventional materials.

### ***Flake Composites***

Flake composites are still in the development stage. Recently they have received considerable attention for structural use where



two-dimensional elements are preferable. The considerable overlap between flakes in the composite can result in an effective barrier against fluid penetration into the matrix as well as reduce the danger of mechanical penetration.

When flakes are embedded in a matrix and made parallel to one another in a plane, they give uniform properties to the composite; however, parallel orientation is difficult to achieve. Flake composites also have special properties due, in part, to their flat shape. They can be packed more tightly than other shapes, thus providing a high percentage of reinforcing material for a given cross section.

Although flakes have good bulk-handling qualities and are relatively inexpensive to produce, desired shapes and sizes are often difficult to achieve. Most metal flakes are aluminum; some silver is used because other metal flakes are difficult to produce. Aluminum flakes can reflect heat, provide a series of protective coatings, and give a metallic luster to coatings and plastic moldings.

Silver flakes are more applicable for composites when high conductivity is needed. For electrical conductivity, flake-to-flake contact is necessary, but this is not easy to achieve without losing good bonding qualities. Silver-flake composites are used for conductive-coating electrical heating elements in equipment subject to low-temperature environments.

Though nonmetallic flakes include both mica and glass, mica is more widely used because of its good heat resistance and dielectric qualities. Mica flakes are not as versatile as glass flakes for moisture barrier and structural applications. With certain binders, however, they can provide a good hermetic seal with vacuum tightness and high-temperature properties. If contoured shapes are desired, flakes can be laminated in several layers, bonded with a resin (such as 5 percent shellac, epoxy, or alkyd), heated until the composite softens, and molded into various shapes.

Glass-flake composites with special strength qualities have been proposed for a variety of structural and nonstructural applications. Less expensive than glass fibers, these flakes have dielectric strength, high heat resistance, and offer a high bending modulus because they are free to bend in only one plane. Structural uses of glass-flake composites include aircraft and missile radomes, battery cases, instrument cases, rocket fins, and rocket motor cases.

Once fabricating difficulties are solved, flake composites will offer attractive possibilities.

As in the case of particulate composites, flakes can be treated as inclusions, and the basic theory of stress concentrations can be applied for general analysis. The practical utility of this approach, however, is extremely limited.

### *Filled Composites*

A filled composite is an open or skeletal matrix containing another material that remains a separate constituent. Both materials and structure can be varied to give a wide range of physical and mechanical properties. Some examples (ref. 5) of what can be done are: (1) improve the function characteristic, (2) increase the strength and ductility of porous metals, (3) prevent leakage of fluids and gases in porous metals, (4) improve the performance of electric contacts, and (5) provide high-temperature properties not obtainable with monolithic materials.

The skeleton may be a solid or a fluid becoming solid during manufacture. The filler may be fluid when introduced into the void structure and may either remain liquid or solidify by cooling. The fluid may be introduced into the structure by a carrier that may be removed, evaporated, or polymerized.

Although the concept of filled composites is old, the full potential of this type of composite was not apparent until the space age. Previously, this technique was used to prevent seepage by filling voids in one material with another. Currently, two types of filled composites are available: (1) porous, or spongelike, structure, and (2) cellular, or honeycomb, structure. Metal, paper, or wax honeycomb filled with a ceramic material; silica honeycomb filled with a ceramic material; and silica honeycomb filled with fiber-reinforced epoxy and silicone rubber are examples of filled cellular composites. Examples of filled porous composites include tungsten or molybdenum impregnated with copper or silver; plastic filled with porous metal such as aluminum, magnesium, or ferrous castings; TFE fluorocarbon and lead-filled bronze; graphite impregnated with lubricating oils; and resin impregnated with aluminum, ceramic, or zirconia foams.

The range of possible uses of filled composites is not fully known. Two current applications are a steel matrix composite containing titanium carbide for making dies, gages, and punches; and a filled honeycomb composite with random network filler metal (ref. 6) for high-temperature aerospace applications.

### *Laminar Composites*

Laminar, or layered, composites are probably the oldest of all the composites; they account for the greatest volume of use and are produced in the greatest variety. They differ in material, form, and/or orientation, as shown in figure 7. Although the great variety of possible combinations makes generalization difficult, we can say that (1) each layer of a laminar composite may perform a separate and distinct function, (2) properties may vary from one side of the composite to

the other, and (3) properties of laminar composites tend to be anisotropic.

Many laminar composites are designed to provide characteristics other than strength; for example, improved appearance, protection against corrosion and/or high temperatures, and adjustment for size limitations.

Some types of laminates are: precoated and preplated materials, clad metals, plastic-based laminates, laminated glass, and laminated nylon fabrics. Specific examples of these laminates are alclads; nickel-plated steel for flashlight cases; electrogalvanized steel for roofings; aluminum-clad uranium, molybdenum-clad aluminum (copper, gold, or lead), and silver-clad aluminum for weave guides; lead-clad steel for radiation shielding; glass-nylon fabric for personnel armor; glass-plastic (usually containing two or more layers of glass sheet and one or more layers of polyvinyl butyrol) for safety glass; laminated layers of transparent plastic for light filters; and asbestos or other mineral-based fabrics laminated with silicone matrices for heavy-duty electrical and high-temperature application.

The analysis of laminates can be lengthy and arduous because of the possible combinations of several different isotropic and/or anisotropic materials. Stress distributions depend on the composition and orientation of the individual layers. A laminate may be isotropic, but if its constituents exhibit different moduli of elasticity or different Poisson's ratios, appreciable nonuniform shear stresses may develop and lead to delamination or other serious problems. Consequently, laminates composed of mixed isotropic and anisotropic materials present a difficult analytical problem.

Stress analysis of a laminate subjected to external loads is based on the fundamental assumption that at any point the deformation of all the constituents is the same; that is, strains are equal. Thus, a laminate made of isotropic materials having the same elastic constants behaves like a solid mass of the same material. If the elastic constants are different, the stresses in the constituents are proportional to the elastic constants (elastic or shear moduli). However, even in directions in which no load is applied, stresses in the individual layers may be produced by different transverse contractions or expansions because of different Poisson's ratios; these tendencies are prevented by the bond between layers. Both transverse shears are produced as well as shear in the bond between layers. When isotropic and orthotropic materials are combined, large differential stresses in the laminate as well as large shears in the bonds between layers may develop. Changes in temperature may similarly cause complex stresses (ref. 6). Furthermore, when a layer of material is thin, as in most laminates, the thickness direction is often ignored; even an orthotropic material is treated

as a substance that has two elastic moduli, one shear modulus, and two Poisson's ratios associated with the natural axes. For analytical approaches to laminates, the reader is referred to Tsai et al. (refs. 12 and 15), Dong et al. (ref. 16), and the bibliography.

### **Sandwich Construction**

Sandwich construction<sup>2</sup> is a special kind of laminate consisting of a thick core of weak, lightweight material sandwiched between two thin layers (called "face sheets") of strong material (fig. 7). This is done to improve structural strength without a corresponding increase in weight; that is, to produce high strength-to-weight ratios. The choice of face sheet and core materials depends heavily on the performance of the materials in the intended operational environment.

Sandwich composites are often compared to an I-beam with a high section modulus. Because of the separation of the core, face sheets can develop very high bending stresses. The core stabilizes the face sheets and develops the required shear strength. Like the web of a beam, the core carries shear stresses. Unlike the web, however, the core maintains continuous support for the face sheets. The core must be rigid enough perpendicularly to the face sheets to prevent crushing, and its shear rigidity must be sufficient to prevent appreciable shearing deformations. Although a sandwich composite never has a shearing rigidity as great as that of a solid piece of face-sheet material, very stiff and light structures can be made from properly designed sandwich composites.

A useful classification of sandwich composites according to their core properties by respective direction is shown in figure 8 (from ref. 17). To see the core effect upon sandwich strength, let us consider the honeycomb-core and the truss-core sandwich composite. The honeycomb sandwich has a ratio of shear rigidities in the  $xz$  and  $yz$

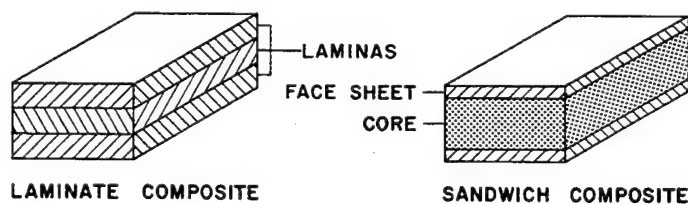


FIGURE 7.—Laminar composites.

<sup>2</sup> The ASTM definition: A structural sandwich is a construction combining alternating, dissimilar simple or composite materials, assembled and intimately fixed in relation to each other so as to use the properties of each for specific structural advantages in the whole assembly.

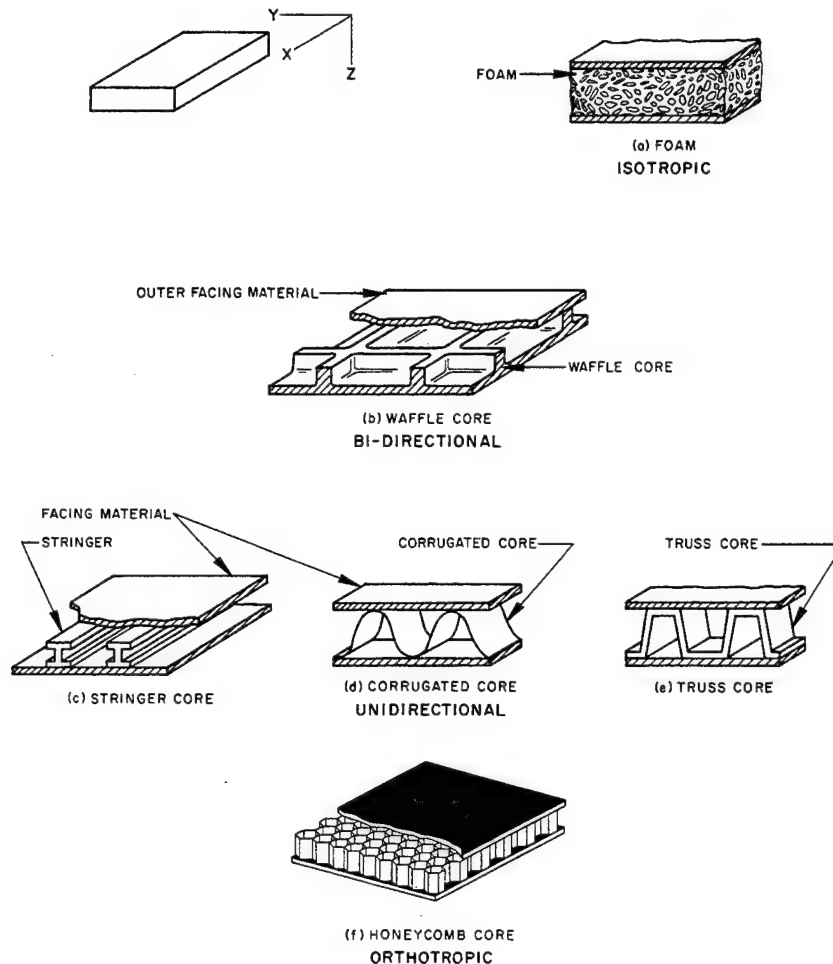
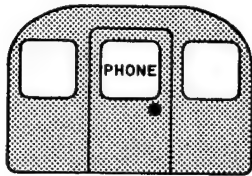


FIGURE 8.—Typical sandwich constructions.

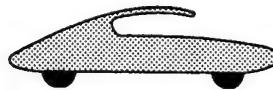
planes of approximately  $2\frac{1}{2}$  to 1. The face sheets carry in-plane compressive and tensile loads, whereas the core stabilizes the sheets and builds up the sandwich section. The truss-core sandwich has a shear rigidity ratio of approximately 20 to 1. It can carry axial loads in the direction of the core orientation as well as perform its primary function of stabilizing the face sheets and building up the sandwich section.

### CURRENT APPLICATIONS

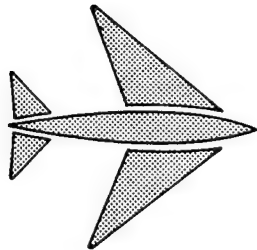
Figures 9 (from ref. 11), 10, 11, and 12 suggest a few of the many uses for composites. Prior to World War II, sandwich construction of birch face sheets and balsa core was used extensively in the British

BUILDING CONSTRUCTION EXAMPLES

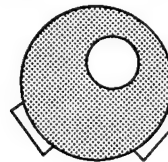
HOMES  
HIGHWAY TOLL BOOTHS  
TELEPHONE BOOTHS  
GAS STATIONS  
HIGH REST STATIONS

TRANSPORTATION INDUSTRY EXAMPLES

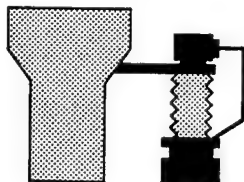
MONORAIL CARS  
ELEVATORS  
TRUCKS AND AUTOMOBILES  
OVER-THE-ROAD TANKERS  
RAILWAY AND SUBWAY CARS

AIRCRAFT INDUSTRY EXAMPLES

AIRCRAFT RADOMES  
LANDING GEAR  
WING AND FUSELAGE STRUCTURES

CHEMICAL PROCESSING EQUIPMENT

REACTION VESSELS  
PIPING  
BULK CHEMICAL TANKS

ELECTRICAL PRODUCTS

SWITCH GEAR  
HIGH VOLTAGE FUSE TUBES  
CIRCUIT BREAKERS  
HIGH VOLTAGE INSULATORS

FIGURE 9.—Potential applications of filament winding. (Courtesy of Materials in Design Engineering.)

De Havilland Mosquito bomber. Since then a continually growing list of sandwich applications in aircraft includes radomes, fuselages and wings, ailerons, floor panels, and storage and pressure tanks. Because of their dielectric properties, plastic, glass, and fabric honeycomb-core sandwiches are being extensively investigated for use in things such as radar housings and microwave transmission windows. The novel, though costly, design of the B-70 features use of machine-fusion welding for joining brazed, heat-treated, steel-

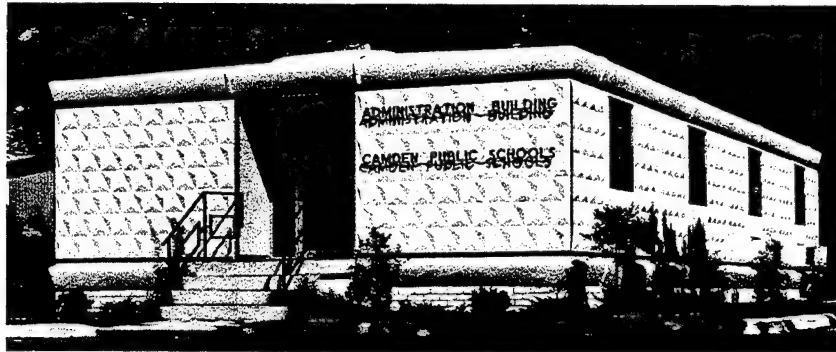


FIGURE 10.—Portable, permanent building, 26 by 56 ft. Exterior is white fiber glass with rigid foam insulation. (Courtesy of Holiday Manufacturing Co., Division of Holiday Inns of America, Inc.)

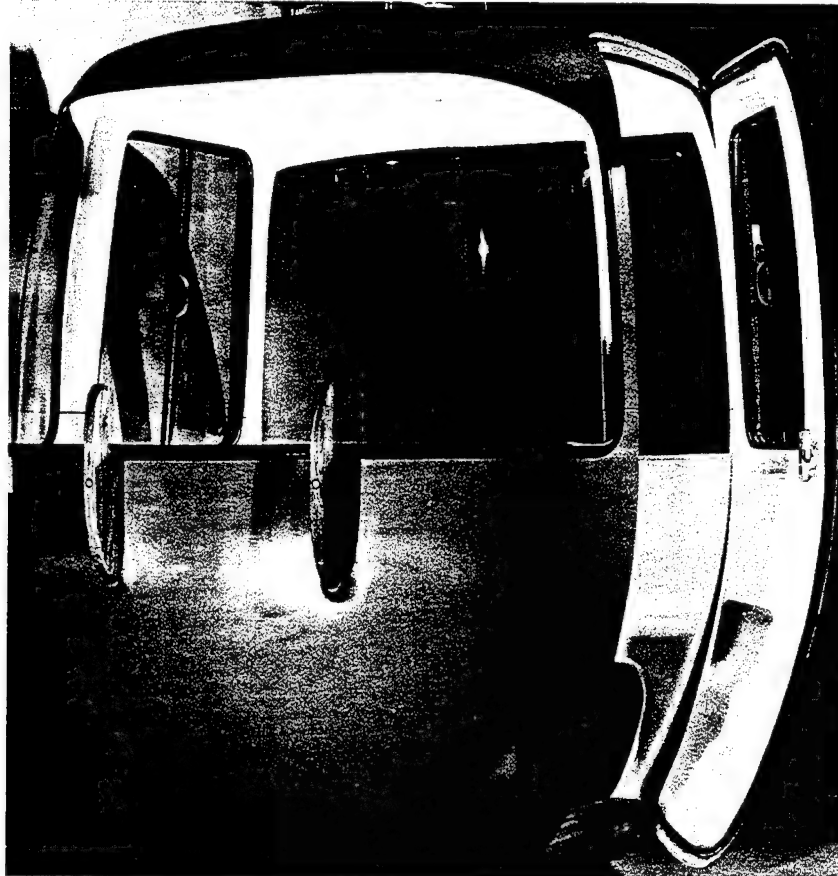


FIGURE 11.—Structural frame for ski lift gondola. (Courtesy of American Cyanamid Co.)

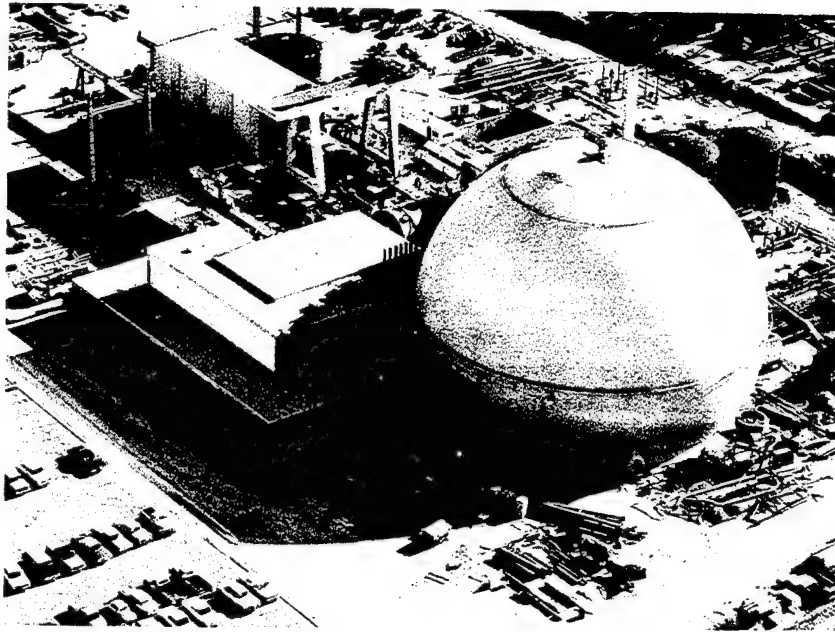


FIGURE 12.—Multilayer wall tank. (*Reprinted from Oil and Gas Journal, Jan. 3, 1967.*)

honeycomb panels into a homogeneous, compression-resistant structure with local stresses and shear flow based on minimum strain energy. In a new space-formed system called "Sunflower," the reflector is of honeycomb construction, having a thin coating of pure aluminum protected by a thin coating of silicon oxide to give the very high reflectivity needed for solar-energy collection. The unit is adaptable as a power source for a wide range of Earth, Moon, Mars, and Venus missions. Thirty panels fold together into a nose-cone package in the launch vehicle.

#### **Building Construction**

Architects use sandwich construction made of a variety of materials for walls, ceilings, floor panels, and roofing. Cores for building materials include urethane foam (slab or foam-in-place), polystyrene foam (board or mold), phenolic foam, phenolic-impregnated paper honeycomb, woven fabrics (glass, nylon, silk, metal, etc.), balsawood, plywood, metal honeycomb, aluminum, and ethylene copolymer foam. Facing sheets can be made from rigid vinyl sheeting (flat or corrugated); glass-reinforced, acrylic-modified polyester; acrylic sheeting; plywood; hardwood; sheet metal (aluminum or steel); glass-reinforced epoxy; decorative laminate; gypsum; asbestos; and poured concrete.



### ***Civil Engineering***

Sandwich construction is now used in bridge decking (stainless-steel truss core), retaining walls, and storage tanks. Structural aluminum honeycomb panels have been used in the construction of the 327-ft-high, 7-million-lb service tower for the Saturn rocket at Cape Kennedy.

### ***Mechanical Design***

Foam-core sandwiches have a promising potential in refrigeration and related fields. Early projects involved panels having an expanded styrene core bonded to styrene sheet faces. More recently, two pilot models have been constructed with anodized aluminum faces which offer important cost advantages in tooling. Presently this construction is used in high-temperature furnaces as well as pressure vessels and tanks, particularly cryogenic tanks.

### ***Damped Structures***

An increasing number of vibration problems must be controlled by damping resonant response. By using a symmetric sandwich panel with a viscoelastic core, various degrees of damping can be achieved, depending on the core material properties, core thickness, and wavelength of the vibration mode.

### ***Marine Structures***

Urethane, expanded styrene, and other types of plastic foam have been used in the construction of small boats. For the same purpose the U.S. Rubber Co. has developed a configuration consisting of an expanded Royalite ABS core with a ply of hard Royalite bonded to each side.

### ***Transportation***

Sandwich construction has great promise for transit cases, floor panels, railway cars, and large transportation carrier panels. It has already been used in the design of carriers for experimental rapid transit systems.

Sandwich construction has undergone analytical and experimental investigations that have resulted in a wealth of data. Plantema (ref. 17) has summarized the theory of strength and stability of sandwich-type structures.

The objective of many of the present investigations is to establish rational guidelines for the design and utilization of composite materials for structural applications. More accurate methods of structural analysis are needed to insure the efficient utilization of composites.

Which composite material should be used in a specific design problem? Unfortunately, there is no simple, direct answer. Many

important factors contribute to the characteristics, properties, and overall performance of composites. To make the best use of composites, we must be able to characterize each desired property and to assess quantitatively the behavior of the resulting composite. To use composites in design, structural engineers need mathematical expressions and models that adequately predict the behavior of composite materials under various load conditions.

We have briefly examined some of the developments and advanced concepts in materials and types of construction stemming from space-vehicle system research and technology programs. As the deficiencies in our knowledge of the behavior of structures are removed, we can come closer to rational design per se. As production reliability and analytical competence are improved, the behavior of structural elements under various loading will become more predictable. Finally, when production costs are reduced, more extensive industrial use of these developments can be expected.

## Selection of Structural Materials and Types

No single known material or construction can meet all the performance requirements of modern structures. Selection of the optimum structural type and material requires systematic evaluation of several possibilities. The primary objective often is to select the most efficient material and configuration for minimum-weight design.

In figure 13 (from ref. 18), materials are plotted according to their strength-to-density ratios. The most commonly used structural materials are clustered in the middle of the figure; those for specialized use (such as fabrics, fiber glass, laminates, and beryllium) are widely dispersed in the lower half of the figure; and the emerging composites containing S-glass, boron, and carbon filaments appear in the upper half. Properties of composite materials are calculated for a planar isotropic layer of filaments in an epoxy-resin matrix. Although this approach is convenient, it is not appropriate for all structures.

Figure 14 (from ref. 19) shows the lightest forms of construction that will carry pressure and bending loads without cylinder buckling or material yielding. The forms considered are filament wound, sandwich, stiffened skin, and simple isotropic walls. Regions to the right of the indicated boundary (slant line) in the chart are those of higher bending moment; that is, those where structures must support compressive and shear stresses efficiently. Consequently, material stiffness is an important requirement. While sandwich constructions of various types have a wide range of application for most loading parameters, the extremes of pressure and bending moment require filament-wound and isotropic constructions, respectively. Filament-wound construction is superior to other structural types for pressure-vessel applications, except when the applied bending-moment index is also large; then the conventional isotropic shell is lighter. For lower or zero internal pressure, all types of wall construction have a range of efficient application, depending on the magnitude of the bending-moment index.

Figure 15 (from ref. 20) is a weight comparison of several types of construction for cylinders as a function of loading intensity for

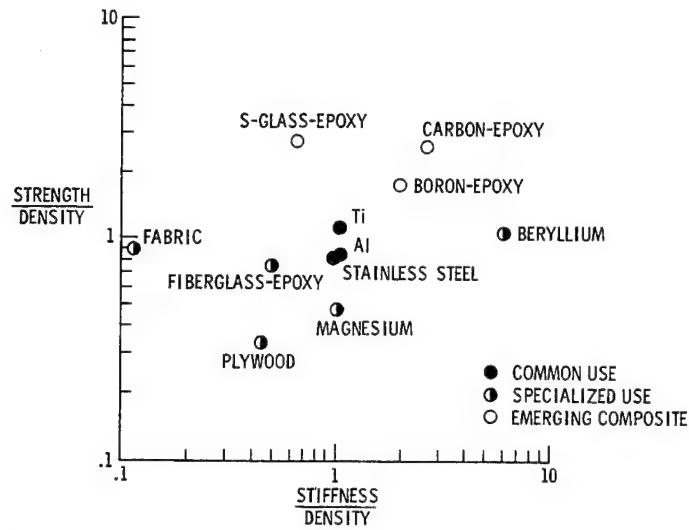


FIGURE 13.—Materials comparison. (Reprinted from *Proceedings AIAA/ASME 7th Structures and Materials Conference*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

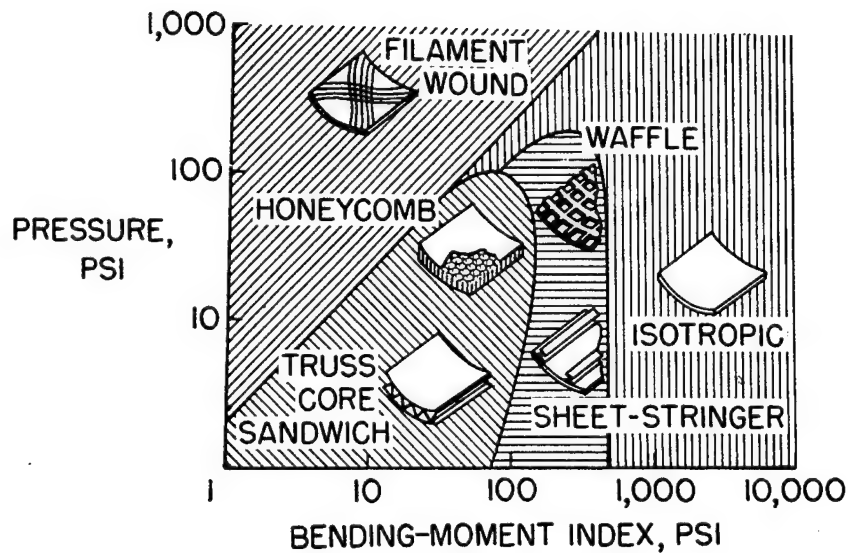


FIGURE 14.—Least-weight construction regimes. (Reprinted from *Astronautics and Aeronautics*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

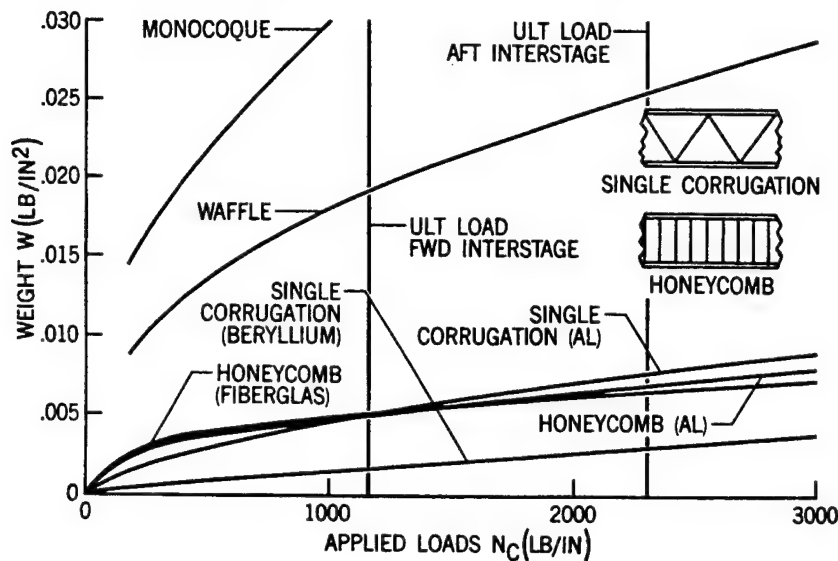


FIGURE 15.—Comparison of minimum-weight designs for structural systems and materials in an axially loaded cylinder.

designs involving combined axial compression and bending. Figure 16 (from ref. 21) presents data for 10-ft-diameter cylinders with local compression load capability of 400–1600 lb/in., which is within the appropriate range for the early orbiting laboratories. Four types of construction were considered: sheet-stringer (stiffened skin), waffle, corrugated truss-core sandwich, and honeycomb sandwich. A preliminary estimate of wall gages required for these load levels indicated that integrally milled waffle plate could be eliminated from the study. In theoretical axial strength alone, the honeycomb wall is the lightest, followed by truss core and sheet stringer. Since weight comparisons are very close, they are very sensitive to design ingenuity.

Figure 17 (from ref. 22) shows a comparison of the structural index curves for variously constructed cylindrical shells subject to axial compression. The curves include isotropic, Z-stiffened, waffle, truss-core sandwich, and multiwall sandwich cylinders. The multiwall sandwich consists of layers of dimpled and flat sheets welded at the crests of the dimpled sheets. Structural index curves for flat plates of various construction subjected to edge compression are shown in figure 18 (from ref. 22). The curves shown in both figures are based on formulae given in reference 22 and citations made in this reference. The curves

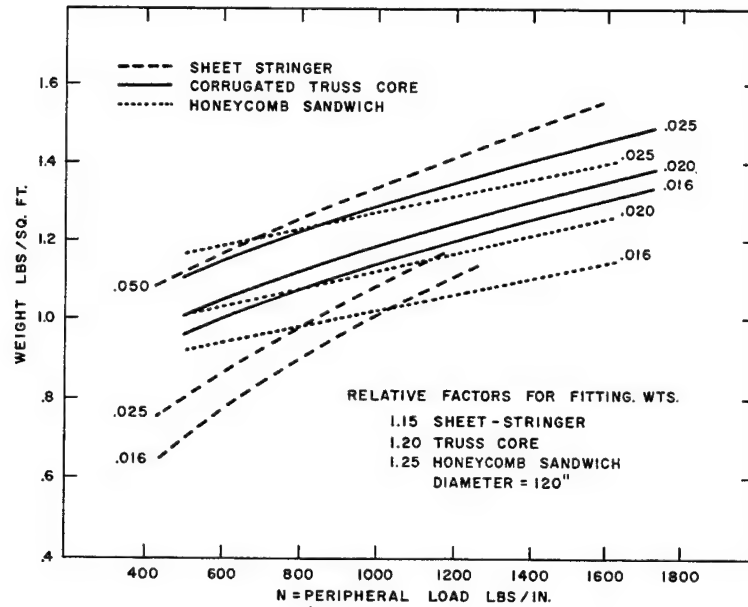


FIGURE 16.—Compartment wall weights.

$$\frac{E}{\sigma_{cy}} = 115 \quad \text{and} \quad \frac{E}{\sigma_{cy}} = 153$$

are related to the original report and are included here merely for reference.  $P_1$  is compressive load per unit width;  $N_x$ , compressive buckling load per unit circumference in the longitudinal direction;  $b$ , width of plate;  $R$ , radius of cylinder;  $\bar{t}$ , equivalent thickness of material;  $E$ , modulus of elasticity;  $\rho_c$ , density of core;  $\rho_s$ , density of face sheets;  $\eta$ , plasticity reduction factor; and  $\sigma_{cy}$ , compressive yield stress.

One of the major reasons for combining inorganic fibers and inorganic matrices is to achieve high-temperature performance not possible with organic materials. One of the new and promising composites under investigation is metal reinforced with alumina whiskers. Figure 19 (from ref. 5) shows the tensile strength of various classes of whisker-reinforced materials at elevated temperatures. So far, strengthening metals with whiskers at elevated temperatures has only been demonstrated in the laboratory.

Theoretical whisker-plastic laminates are compared with present bidirectional materials in figure 20 (from ref. 14). The calculated strength is based on the assumption that the whiskers are nonwoven, biaxial, felt impregnated with epoxy resin, and high pressure molded.

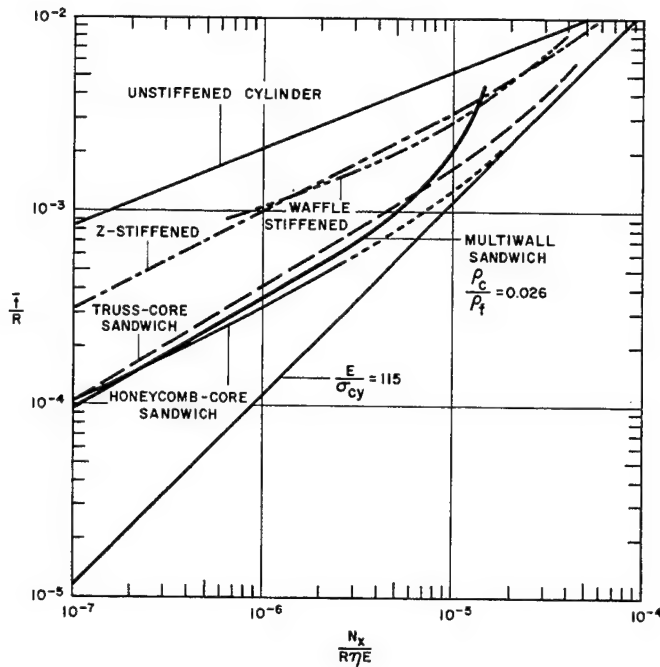


FIGURE 17.—Structural index curves for cylindrical shells of various constructions subjected to axial compression.

Under these assumptions, the curves show that the whisker composite would offer an order-of-magnitude increase in strength.

Figure 21 (from ref. 23) summarizes the evaluation for pressure vessels. On the horizontal scale, the uniaxial material tensile-strength-weight ratios are indicated. For glass filaments, this ratio is based on the strength of the rovings. The vertical scale represents the uniaxial structural strength-to-weight ratios that can be achieved by application of the best current technology. Although filamentary composites appear to be superior to monolithic construction, it must be noted that the configuration efficiency coefficient must also be considered when evaluating overall pressure-vessel efficiencies. Consequently, figure 21 does not permit a direct comparison of the relative efficiencies of materials as used in pressure vessels. The values in figure 21 are based on short-time-load applications at room temperature; cryogenic and elevated temperatures and other environmental factors can change the relative efficiencies of monolithic and filamentary materials substantially.

Figure 22 (from ref. 23) depicts the overall efficiencies of monolithic and filamentary materials for membrane-type pressure vessels.

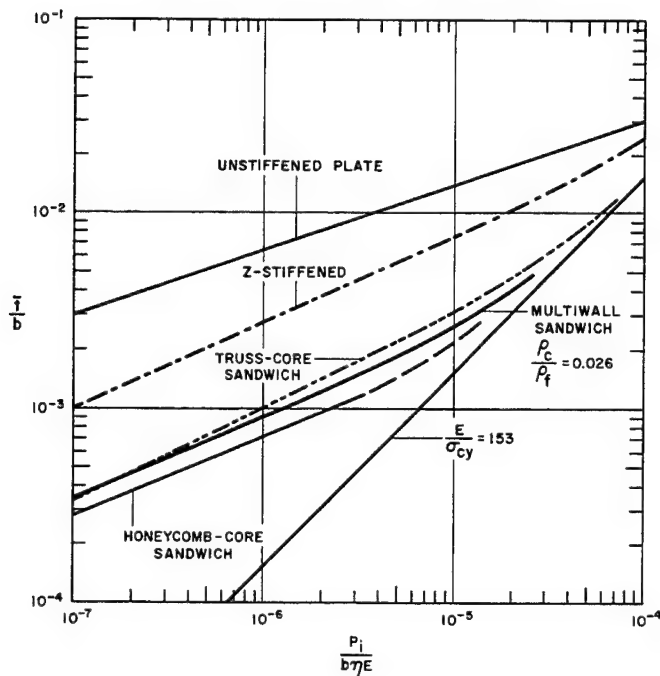


FIGURE 18.—Structural index curves for flat plates of various constructions subjected to edge compression.

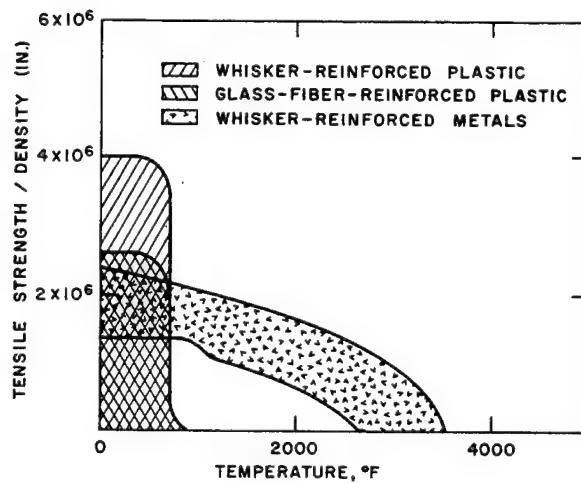


FIGURE 19.—Strength versus temperature for various fiber composites. (Courtesy of Materials in Design Engineering.)



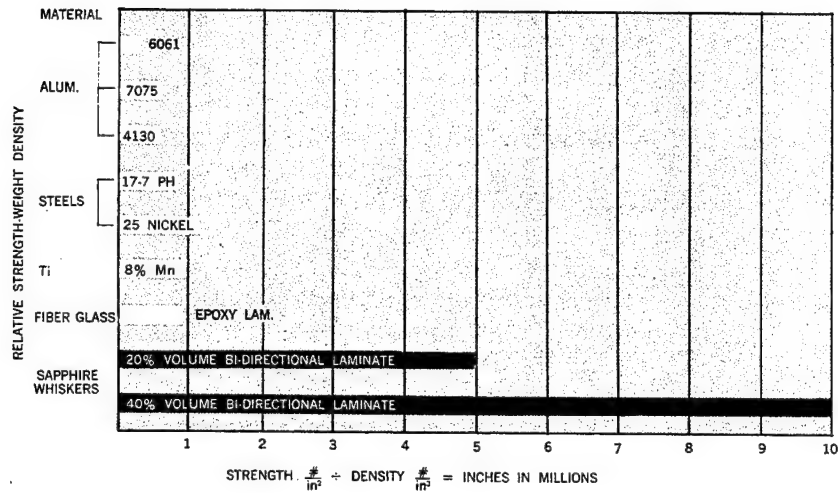


FIGURE 20.—Strength density of various bidirectional structural materials.  
(Reprinted by permission of the publisher, F. D. Thompson Publications, Inc.,  
from Research/Development Magazine, March 1966.)

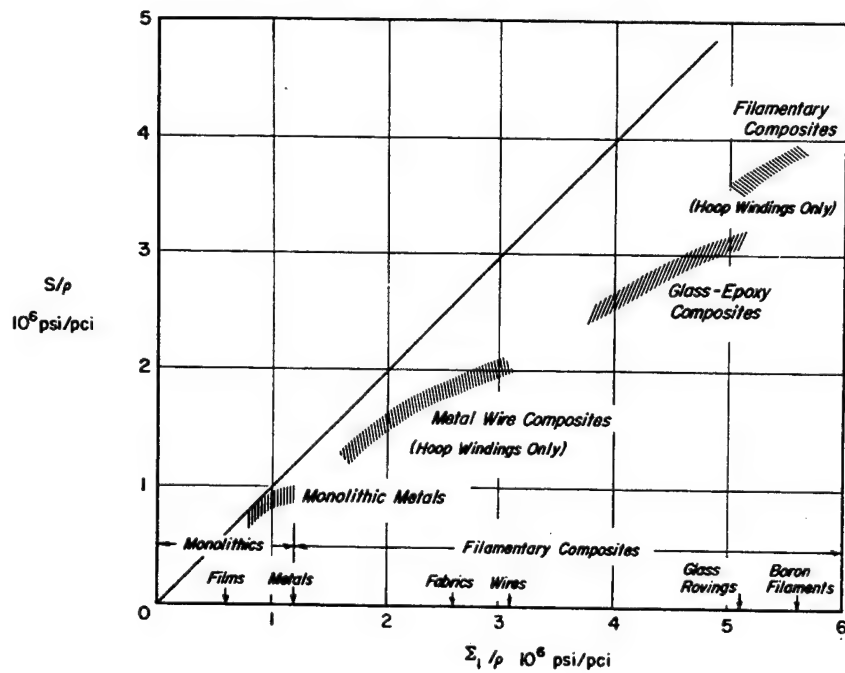


FIGURE 21.—Comparative structural efficiencies of various materials in pressure  
vessel applications at room temperature.

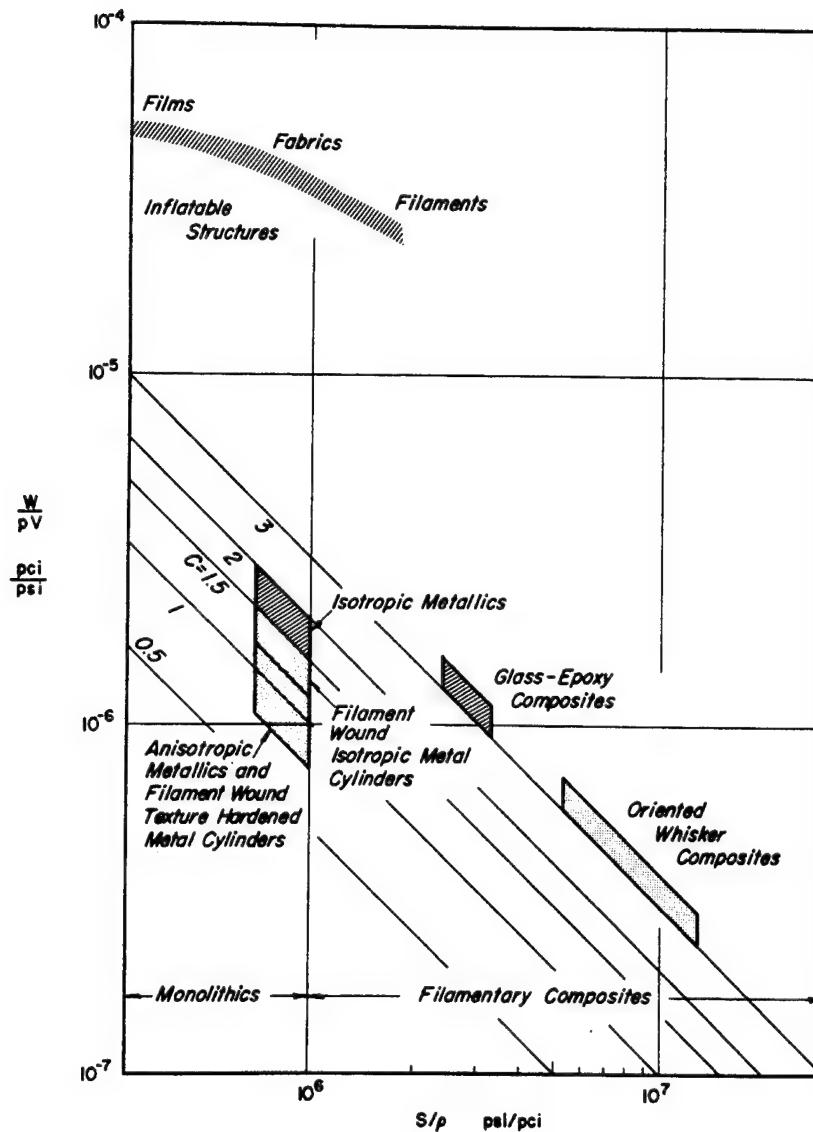


FIGURE 22.—Overall membrane efficiencies of pressure vessels at room temperature.

It is based on two efficiency factors: (1) structural efficiency coefficient,  $C$ , and (2) material efficiency parameters,  $(S/p)$ ; where  $C$  is a nondimensional function of the configuration and material-failure law;  $\rho$ , the density in pci; and  $S$ , the structural strength in psi. The crosshatched regions in figure 22 represent materials that have been

utilized in full-scale aerospace-production components. One should remember that the aerospace environment encompasses temperatures other than room temperature, on which figure 22 is based.

The data of Brewer and Jeppeson (ref. 24) indicate that inflatable structures as a class are inherently much less efficient than metallic and glass-epoxy composites. Isotropic metallics are not as efficient as glass-epoxy composites, when properties are compared at room temperature. Under the best circumstances for each, a weight-saving potential of approximately one-third can be attained with the glass-epoxy composite.

For other materials concepts that have not, as yet, reached the aerospace production stage, filament-wound isotropic metal cylinders represent an inherent improvement over monolithic isotropic metallics. At room temperature, however, the glass-epoxy composites still appear to have an advantage. On the other hand, anisotropic metals, as opposed to currently used materials, can represent a significant weight-saving potential. This potential depends strongly on the degree of anisotropy that can be achieved with high-strength metals and the configuration of the pressure vessel. This is also true for filament-wound, texture-hardened metal cylinders.

An important improvement in overall efficiency appears possible with oriented whisker composites. However, on the basis of the analysis used herein, the potential of such composites appears to be far less dramatic than predicted by Hoffman (ref. 25). In fact, only the low-density whiskers, such as graphite and aluminum oxide, appear to be attractive when used in the form of oriented whisker composites.

In selecting a material and design for specific performance requirements, each configuration (and each part of the configuration) must be examined and analyzed to provide the best possible structure. Ultimately, materials design will be integrated into structural design as an added dimension. Since the selection of the configuration requires consideration of the environment, rigidity requirements, fabricability, smoothness, and reliability, a detailed analysis is needed to provide a valid basis for selection. In chapter 5, we will consider the interplay among design, structures, and materials as well as the general aspects of design synthesis and optimization.

## Structural Concepts and Applications

In chapter 2, we emphasized advances in strengthening materials for structural applications that have resulted in part from aerospace requirements. We now turn to some of the recent developments in structural concepts, their uses, and general types of construction.

### LAMINATION

Structural types may be used singly or in combinations, depending on the functional requirements of the object to be constructed. For example, for ordinary performance a pressure vessel may be made from a monolithic material, but when weight is a critical factor, it can be made from a filament-wound design. As components of liquid-hydrogen flight vehicles, vessels must withstand extremely high temperatures for long periods of time without serious loss of structural integrity. Composite laminates permit multifunctional constructive systems that have this capability. This new structural concept involves layers of either monolithic or composite materials. Three examples developed for application in reusable structures are: hot monocoque, insulated, and multiwall designs.

#### ***Hot Monocoque***

Figure 23 shows a hot-monocoque structure for a hydrogen tank which operates near equilibrium temperature and supports applied load. The interior systems, consisting of an aluminum waffle-plate tank with reinforcing rings, are isolated from the exterior load-bearing, or primary, structure by insulation and by a carbon dioxide purge system. Panels of fibrous insulation are bound to the outside, with carbon dioxide filling the voids in the insulation between the tank and the outer structure. The primary structure is a corrugation-stiffened panel of a high-temperature superalloy with transverse rings for additional support.

#### ***Insulated Design***

The insulated structure concept seen in figure 24(a) is composed of a superalloy heat shield for temperatures up to 1800° F, fibrous high-temperature insulation, a primary structure, cryogenic insulation, and a fuel-tank structure. The temperature of the primary structure is

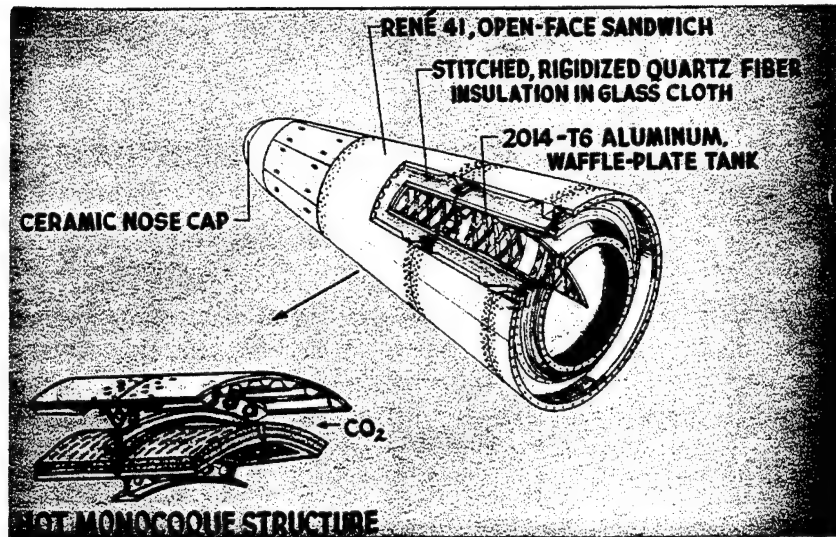


FIGURE 23.—Structural research model.

partly dependent on the thickness of the cryogenic and fibrous insulations. This type of construction requires essentially three leaktight shells: (1) the internal hydrogen tank, (2) the primary structure which precludes liquefaction of air that enters the cryogenic insulation area, and (3) the heat shield which prevents trapping and freezing of moisture within the fibrous insulation area.

#### **Multiwall Design**

The multiwall design, shown in figure 24(b), is unique because the thermal-protection and load-carrying functions are performed by one integral component. The design consists of a sandwich of alternating layers of flat and dimpled sheets joined by welds at the dimples. The insulating effect is produced by the multilayer reflective sheets when the spaces between these layers are evacuated. The inner layers form both the primary load-carrying structure and the tank wall. Because large temperature differences through the wall thickness are a major problem, the potential of this concept is limited, first, by manufacturing difficulties and, second, by possible thermal stresses inherent in its complex design. A multifunctional, multilayer laminate, nevertheless, has been successfully used in a rocket-nozzle design to withstand 6800° F.

Laminates have also been used in filters, printed circuitboards, and skis.

Several layers of felts or other fibrous materials can be bonded by

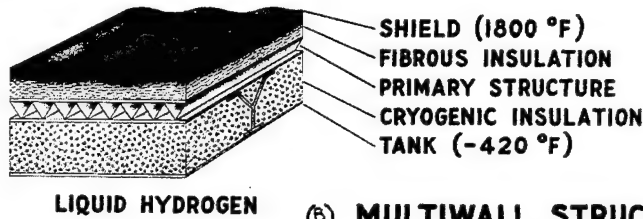
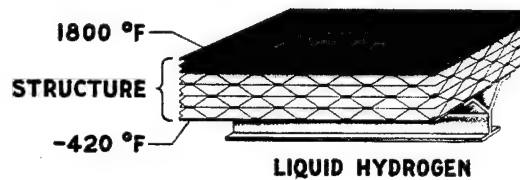
**(A) INSULATED STRUCTURE****(B) MULTIWALL STRUCTURE**

FIGURE 24.—Cryogenic tankage for hypersonic aircraft.

interlocking the fibers. Furthermore, layers of different fiber systems with varying pore sizes, densities, and thicknesses can be bonded together to form a filter laminate, in which each layer can separate particles by specific sizes. Recently a printed circuitboard consisting of a layer of silicone rubber bonded between two layers of glass-reinforced-epoxy laminates was introduced. The glass-epoxy layers are clad with copper to provide good electrical conductivity; the silicone rubber gives damping power; finally, the glass-epoxy adds strength, rigidity, and insulating properties.

A new ski design uses a seven-layer laminate shown in figure 25. After a layer of wood-particle board is bonded between two aluminum strips, the aluminum strips are bonded to two strips of high carbon steel to provide camber and flexure. Lastly, cotton fabric layers are applied to the aluminum to increase its bond strength to the wood and the steel. The top, bottom, and sides are each bonded to a layer of phenolic plastic.

Many other design problems can be solved with plastic laminates bonded to organic or inorganic materials. Potential advantages of choosing materials for specific purposes include: better strength-to-weight ratios, increased rigidity and strength for soft sealing materials, dimensional stability over a wide temperature range, improved bearing surfaces and fabrication characteristics, greater range of frictional and electrical characteristics, higher resistance to corrosion and chemicals, and reduced costs.

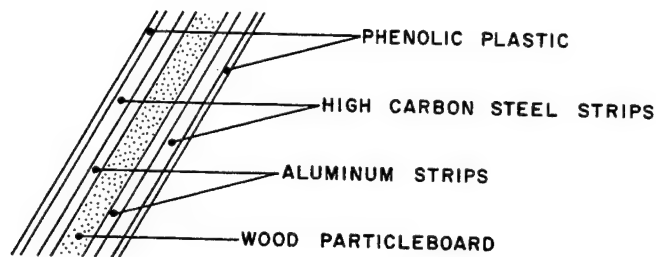


FIGURE 25.—Cross section of ski.

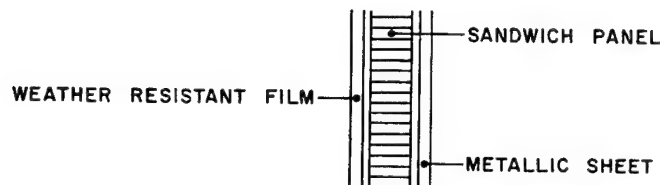


FIGURE 26.—Wall section.

Composite laminates have been shown to provide almost limitless design possibilities and versatility. An example of a future commercial application is a structural wall (fig. 26) that can be used widely in the construction industry. From left to right, the layers consist of (1) a film which serves both for weather protection and decoration, and (2) a sandwich panel, bonded to a metallic sheet, for load support and insulation; this, in turn, provides for radiant heating and cooling. Another layer of fluorescent material could be added for lighting (p. 118 of ref. 5).

### FILAMENT-OVERWRAPPED PRESSURE VESSELS

Although glass-fiber composites are excellent for many structural applications, their use in pressure-vessel applications is limited. Johns and Kaufman (ref. 26) of NASA have described cylindrical cryogenic pressure vessels made by wrapping glass fibers around a metallic vessel in such a way that the metal acts as an impervious liner as well as supports a large part of the pressure load. In overcoming the yield strain difference between the glass fibers and metal, the glass fibers may be prestressed to put the metal into precompression.

The prestressing problem must be carefully considered. Although prestressing by pretensioning is generally desirable during winding, pressurization may be necessary, depending on the amount of prestressing required. To prevent damage during winding, a number of

glass fibers, such as S-HTS glass, can be wound at about 25 percent of their ultimate load. If the vessel is to be used in either high-temperature or cryogenic environments, the difference in the thermal expansion coefficient of the filamentary and the metallic materials must be taken into account in prestressing. For example, an aluminum cylinder wrapped with S-HTS glass at room temperature with near-maximum prestrain will lose most of the prestress at cryogenic temperatures. Cases of this type require special winding techniques to obtain the necessary prestrains without filamentary damage.

In the course of the work described by Johns and Kaufman (ref. 26), aluminum cylinders were wound with sufficient glass filament to carry about half the hoop load at burst pressure, as based on uniaxial tensile properties. Because the metal and filaments reach their ultimate strengths simultaneously, this amount of fiber-glass-reinforced plastic is referred to as optimum. These cylinders were designed to have a one-to-one biaxial stress field at burst pressure, with the filaments being uniaxially wound.

A number of small overwrapped cylindrical pressure vessels were tested to burst. (See fig. 27 (a), (b), (c), and (d), from ref. 26.) The 2014-T6 aluminum tubing was wrapped with S-HTS glass impregnated with epoxy resin to form a layer of fiber-glass-reinforced plastic. Most of the vessels were pressurized to burst. In the optimum design, the metal is designed to be in a one-to-one stress field at burst pressure, where the failure orientation in the metal is not readily predictable. The fracture usually originates in the metal; the failure is either circumferential or longitudinal, or often both. When less than the optimum amount of glass has been used, the fractures seem to originate in the glass almost as often as in the metal.

When cylinders having optimum amounts of glass were tested at room temperature, as shown in figure 27(a), some of them failed without the glass breaking because of circumferential stress in the metal. In these cases, the resin had crazed during straining, allowing the pressure to escape when the aluminum failed and leaving the glass intact. When the tests were repeated with liquid nitrogen, the aluminum failed because of longitudinal stresses, as shown in figure 27(b). In some cases, the failure produced a sawtooth pattern; in others, a smooth pattern. Tests conducted on cylinders in liquid nitrogen with 90 percent of the optimum amount of glass indicated that the glass ruptured first, allowing the aluminum to bulge because of plastic flow. (See fig. 27(c).) Failures during tests conducted on both types of vessels in liquid hydrogen were catastrophic, as shown in figure 27(d). In similar experiments 2014-T6 aluminum cylinders wrapped with S-HTS glass proved to be as much as 50 percent more efficient than homogeneous 2014-T6 aluminum cylindrical pressure



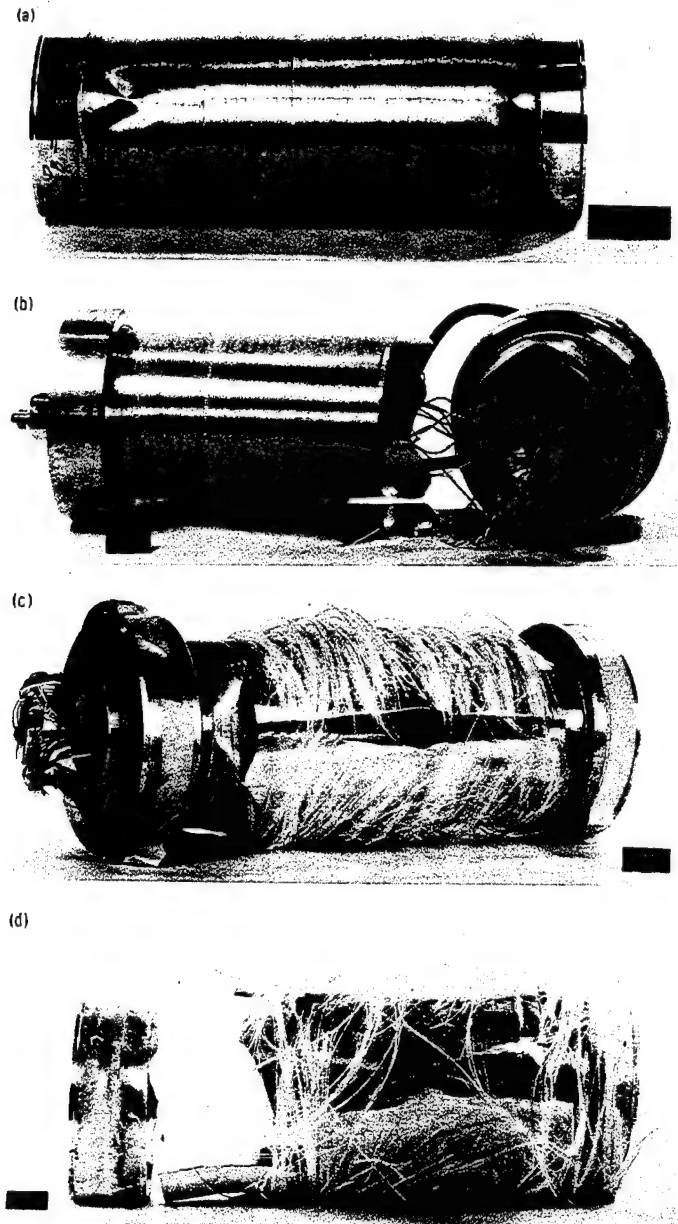


FIGURE 27.—Failures of 2014-T6 aluminum pressure vessels overwrapped with S-HTS glass: (a) 70° F, optimum overwrap; (b) -320° F, optimum overwrap; (c) -320° F, 90 percent of optimum overwrap; and (d) -423° F, optimum overwrap. (Reprinted from *Proceedings, AIAA/ASME 7th Structures and Materials Conference*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

vessels, and consequently, more efficient than spherical pressure vessels.

The greatest potential use for overwrapped tanks is as high-pressure containers since minimum thickness requirements based on fabrication and handling considerations usually predominate in low-pressure applications.

### SEGMENTATION OF TANKS

Because enormous propellant tanks are needed for large launch vehicles, engineers have made radical changes in tank shapes and construction. The size of elliptical bulkheads such as those used on conventional tanks became critical because of the length of the launch vehicle. The usual bulkheads would not only add to the length but also create stability problems due to the increased skirt length, which is the peripheral section between tanks. When diameter increases, the thrust load and geometry require such increased skin gages and stiffener sizes for the skirts that machined integral panels are eliminated.

To solve this problem, NASA-Marshall investigated new concepts for large vehicle propellant tanks. Of these concepts, three are described below: (1) the multicell tank, (2) the semitoroidal tank, and (3) the flat-bulkhead tank.

Following a suggestion made by Professor Oberth some 40 years ago, NASA-Marshall conducted a detailed study of segmented designs which have been used quite successfully in large storage tanks for many years. One type of segmented tank is the integral cluster, scalloped, or multicell configuration.<sup>1</sup> The use of the multicell configuration instead of the conventional cylindrical pressure vessel is an innovation developed for launch systems. A 10-lobe version of the multicell design (ref. 27), shown in figure 28, is composed of thin-walled, partial-circular, cylindrical shells and radial webs. The partial cylinders that form the tank periphery and the radial webs may be of unstiffened, stiffened, or sandwich construction. The radial webs extend from a center tube to the juncture of two outer wall sections and then longitudinally between cell and closure bulkheads. Bulkheads are partial cones connected to the partial cylinders by spherical sectors. Extended and partial Y-sections are used as attachments for cylinder-web-bulkhead junctures and cylinder-spherical skirt junctures along the periphery of the cross section, respectively.

One advantage of the multicell configuration over the conventional

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<sup>1</sup> The multicell configuration is no longer an isolated concept, but is now considered to be a tank with low-profile bulkheads.

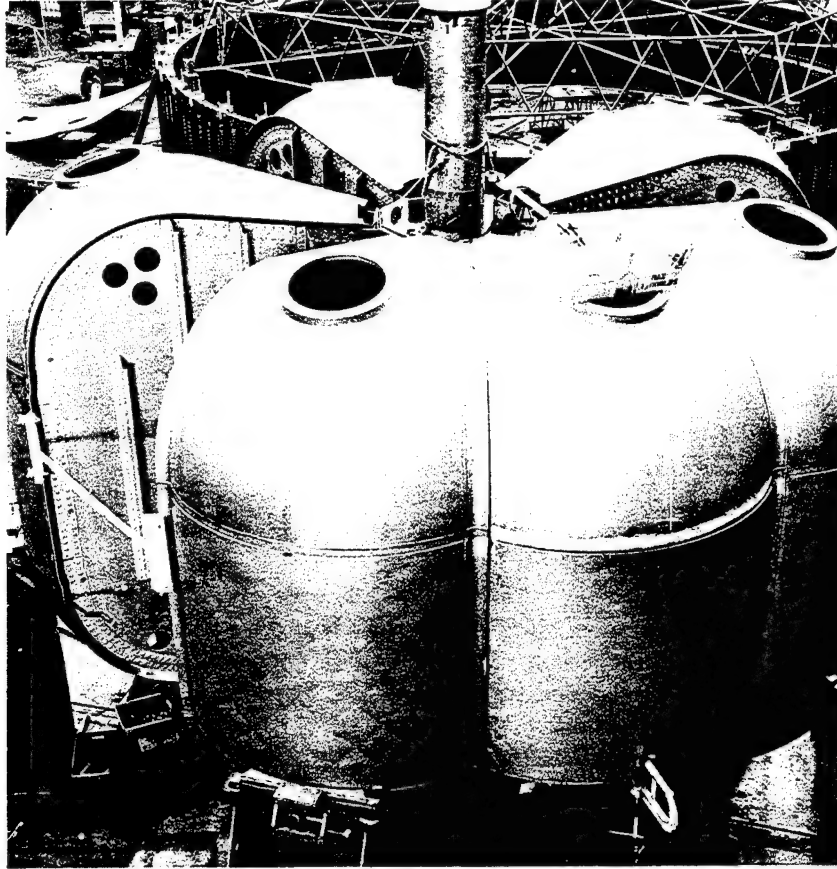


FIGURE 28.—Ten-lobe multicell tank.

pressure vessel design is the reduction in bulkhead depth. As figure 29(a) shows, the bulkhead of the multicell structure is relatively flat. The multicell design not only permits a reduction in overall missile length by decreasing the length of the tanks but greatly shortens the space between the tanks themselves. The radial webs, used most efficiently as part of the basic structure, eliminate the need for baffles to reduce sloshing. The multicell construction provides a flexibility that no other configuration can offer; namely, it distributes basically needed material for a given pressure vessel into both the outer shell and the internal tension wall system. Furthermore, it offers flexibility in selecting tank diameters and bulkhead arrangements and makes it possible to use existing facilities for manufacturing sections of a multicell vehicle. Blumrich (refs. 27, 28, and 29) and Wuensher and Berge (ref. 30) of NASA, among others, have been associated with this launch vehicle design, and their reports include excellent discus-

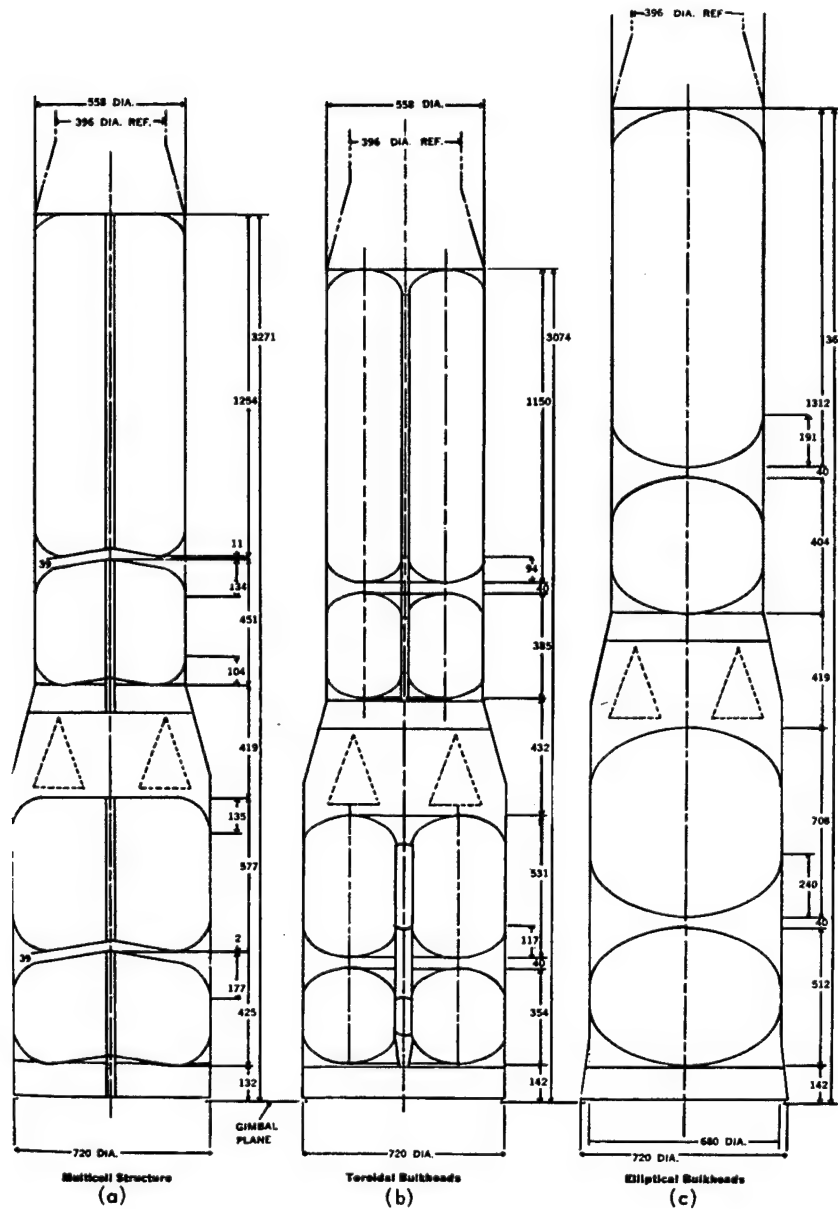


FIGURE 29.—Large-size first- and second-stage structural systems (dimensions in inches). (Reprinted from *Astronautics and Aeronautics*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

sions of the design and development of manufacturing techniques. The analysis of multicell structures also has been treated recently by Blum (ref. 31) and Wilson et al. (ref. 32).

Another principle under investigation is the semitoroidal tank, shown in figure 29(b), in which two ellipses smaller than those in 29(a) form a bulkhead. In figure 29 (from ref. 29), three design concepts are compared: the multicell, semitoroidal, and elliptical bulkhead. Features of the semitoroidal tank are: (1) supports between the tanks and between rear tank and thrust structure; (2) the centerpost which has a diameter determined by the acceptable thickness of the adjacent bulkhead portion; and (3) a connection from the bulkhead to the centerpost. The tank is supported on the thrust structure by the tail section which has either radial beams or at least one member extending through the centerpost of the vehicle to pick up the load. If the material is too thick, it is not possible to make a tangential connection from the elliptical bulkhead to the centerpost. For structural and manufacturing reasons, a conical transition between bulkhead and centerpost seems to be preferable. The advantages of the semitoroidal design include: (1) reduction of stage and vehicle lengths, and (2) elimination of deep elliptical bulkheads because the new design permits the use of separate tanks, with some additional reduction of stage length.

A third principle under investigation is the flat-bulkhead concept (shown in fig. 30 from ref. 27), so named because of the overall appearance of the design. The concept is that of a segmented tank employing several of the principles already discussed under multicell tanks. Further tests are being conducted on a model of the flat-bulkhead concept to determine its structural integrity. Resulting data may be used to compare tank designs and determine preferability.

### ISOTENSOID STRUCTURES

Design problems involving filamentary-matrix construction are simplified if the direction of loading is confined to the principal directions of stress and shear stresses in the matrix are avoided. When shear stresses can be prevented or offset, conditions such as those found in so-called isotensoid structures (ref. 5, p. 126) are produced.

In isotensoid structures, the filaments (in filament-wound structures) are oriented so that they are equally stressed and provide resistance in the principal stress directions in proportion to the magnitude of principal stresses. Because this technique allows circumferential stresses to be twice as great as axial stresses, it is excellent for cylindrical pressure vessels in which about half the fibers

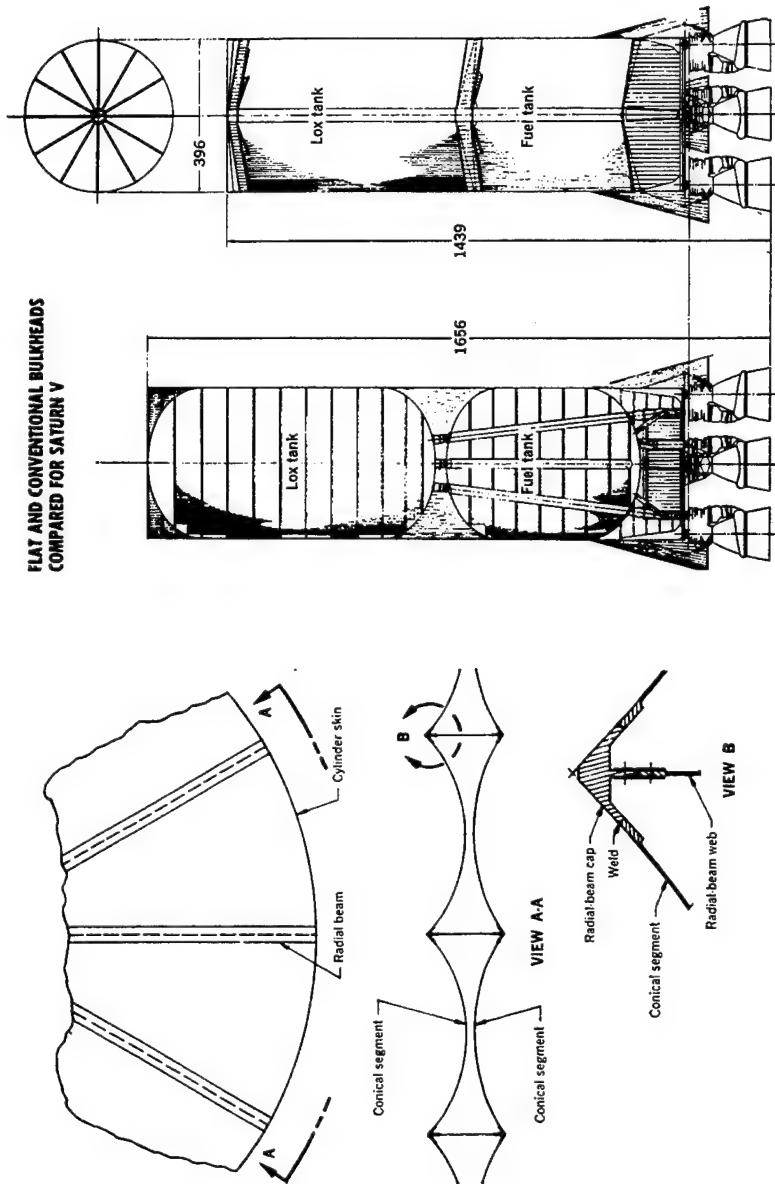


FIGURE 30.—Flat-bulkhead schematic. (Reprinted from *Astronautics and Aeronautics*.) (Courtesy of *American Institute of Aeronautics and Astronautics*.)

are wound axially and half circumferentially. For the preferred helical winding pattern, the filament is wound using an angle to produce its tensile resistance so as to give a desired 2-to-1 ratio when resolved in the two directions of principal stress. Under design conditions, however, such a layer of filament will produce considerable shear stresses in the matrix. Therefore, a second layer is wound in the opposite direction (a reverse wind) to offset the shear stresses.

Comparisons of practical design parameters of the cylindrical and spherical shells have shown that a sphere is a more efficient strength-to-weight ratio pressure vessel. Glass-fiber-reinforced plastics have been found to be the best basic constituents.

The isotenoid design is based on the concept of designing an equal and uniform tension in each fiber. Levenetz (ref. 33) showed that certain modifications of the spherical shape can improve the efficiency of a vessel. He designed the winding pattern of the fibers to maintain unidirectional loading and uniform tension. The geometry of this modified sphere is called oblate spheroid, ovaloid, or ellipsoid, as shown in figure 31. This efficient type of pressure vessel is characterized by a short polar axis and a larger perpendicular equatorial diameter. The head shape is determined by an elliptic integral. The only parameter is the ratio of the central opening to the vessel diameter, which determines the variations of the winding angles, with fibers oriented toward the polar axis. The angle of the fibers with the polar axis depends on the polar openings (end closures).

Composite stresses of 200 000 psi have been reported in rocket cases of this configuration. Recent research at NASA Langley Re-

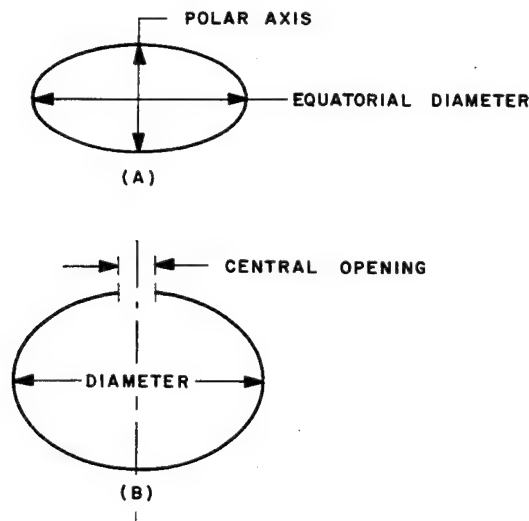


FIGURE 31.—Isotenoid configuration.

search Center (ref. 34) and in industry on the isotensoid concept of filament winding has resulted in extremely high strength-to-weight ratio rocket-motor cases. The determination of the coordinates for isotensoid pressure vessels is given by Zeckel (ref. 35) and in other references.

### TENSION-SHELL CONFIGURATION

The tension-shell configuration, sometimes called the "Langley tension shell," is a reentry vehicle design developed for the Voyager mission to meet minimum weight and high-drag profile requirements. Since high-drag profiles generally have a wide base as compared to axial length, buckling is the usual mode of failure of such a configuration.

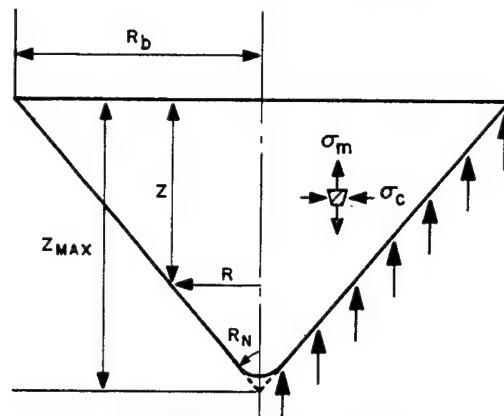
In figure 32 (a) and (b), a conventional design is compared with tension-shell design. In the tension-shell configuration in figure 32 (b), the meridional contour flares outward toward the base. The payload is so attached to the shell that the inertial forces, resulting from the deceleration of the payload, develop an axial tension in the flared portion of the shell. Although the external aerodynamic pressure tends to produce circumferential compression in the shell, this compression is more than compensated for by the circumferential tension produced from a combination of the axial tension and the flared portion. In turn, this tension-shell configuration reduces the buckling tendency of the shell. Nevertheless, the crushing action of aerodynamic forces must be resisted even in the tension shell with a compression-resistant spherical nose segment at the forward end and a compression-resistant ring at the aft section. Papers by Anderson et al. (ref. 36), Halberg (ref. 37), and Levy and Hess (ref. 38) give detailed discussions and analyses of the tension shell.

### TENSION-STRING STRUCTURE

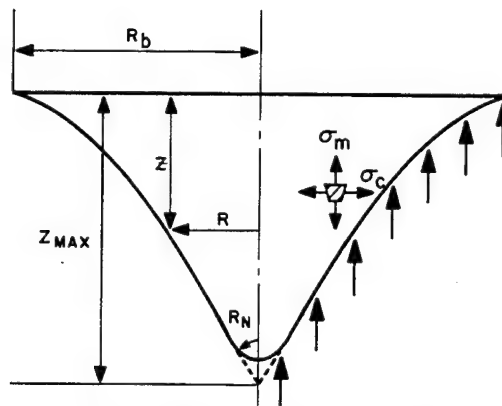
Alai (ref. 39) has reported a new ultralight, high-drag concept called the "tension-string structure," which is a variation of the tension shell. This configuration, lying between that of the sphere cone and a tension shell, is produced by using high-strength filament materials in tension, as shown in figure 33 (from ref. 39). The basic elements are a strong forebody and shield, nose cap, equatorial ring, afterbody, central support, and payload. Figure 34 (from ref. 39) shows a completed model.

A forebody is generated by straight strings in tension, arranged to form a curved (ruled) hyperboloid surface. A specified amount of pretension is applied to each string during winding to assure tension under all loading conditions specified by the mission. The strings





(A) CONVENTIONAL DESIGN



(B) TENSION-SHELL DESIGN

FIGURE 32.—Tension-shell structure.

generate a surface of revolution of negative Gaussian curvature. The finished shape is obtained by applying an elastomeric-shield material over the filaments. The nose cap may be either blunt or pointed, wound along with the forebody, or solidly integrated with the payload. The equatorial ring, one of the primary compression elements, is restrained (bonded) by the forebody and afterbody strings, stabilizing the ring laterally while providing resistance against lower-mode overall buckling.

The afterbody, wound as a continuation of the forebody, is shaped like a truncated hyperboloid. The exact configuration, however, is

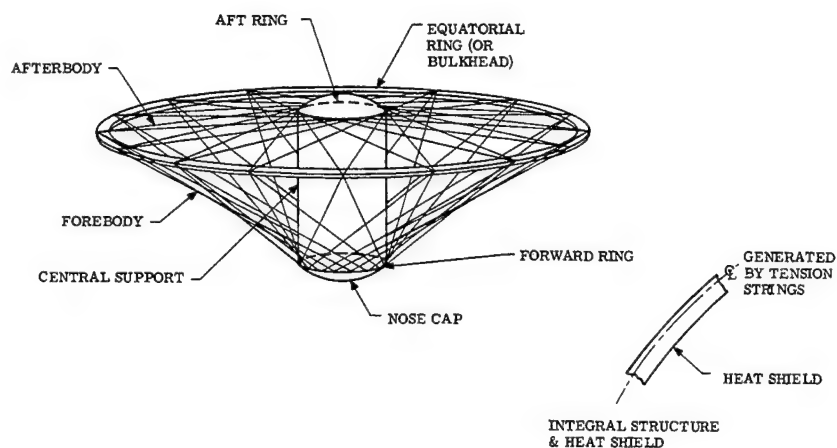


FIGURE 33.—Tension-string structure. (Reprinted from *Proceedings, AIAA/ASME 7th Structures and Materials Conference.*) (Courtesy of American Institute of Aeronautics and Astronautics.)

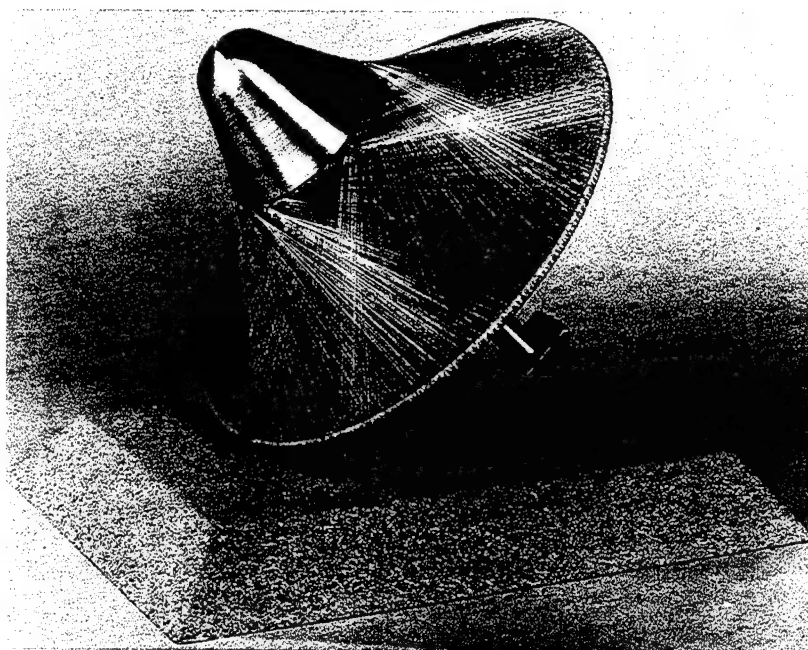


FIGURE 34.—Tension-string model. (Reprinted from *Proceedings, AIAA/ASME 7th Structures and Materials Conference.*) (Courtesy of American Institute of Aeronautics and Astronautics.)

arbitrary and depends ultimately on vehicle requirements rather than on aerodynamic considerations.

The central support, the other primary compression element, may be any shape required. In figure 34 it is shown as a cylinder supporting the payload and fore-and-aft rings, to which forebody and afterbody strings are bonded.

### EXPANDABLE STRUCTURES

The size and mass of missile payloads will always be restricted. At present, there are two approaches to transporting large structures into space in small, lightweight packages. One approach is space construction, requiring prefabricated sections to be launched, rendezvoused, and assembled in space. The second uses a structure that can be expanded from a small to a large volume. Figure 35 (from ref. 40) is a step-by-step illustration of the ejection, erection, and rigidification of an antenna dish. There are four basic types of expandable structures: (1) inflatable, (2) chemically rigidified, (3) unfurlable, and (4) elastic

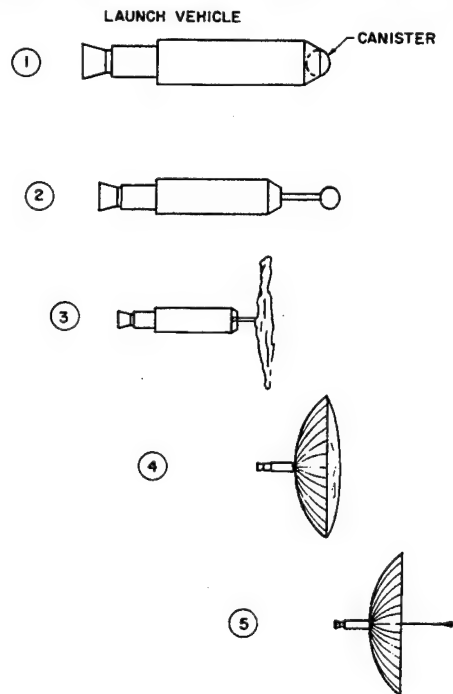


FIGURE 35.—Ejection, erection and rigidification of antenna dish. (Reprinted from *Proceedings, AIAA/ASME 7th Structures and Materials Conference.*) (Courtesy of American Institute of Aeronautics and Astronautics.)

recovery structures. Seven basic techniques (see fig. 36 from ref. 41) are available to the structural engineer to bring about the expansion and rigidification of flexible materials. Brink et al. (ref. 42) and Schuerch and Schindler (ref. 43) have analyzed the foldability of expandable structures.

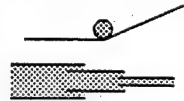
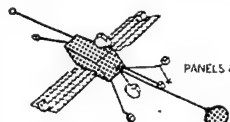
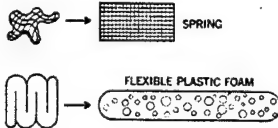
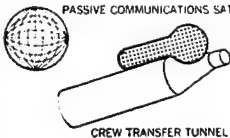

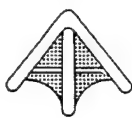
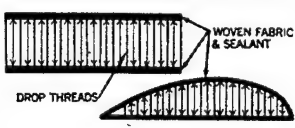
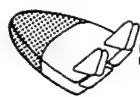
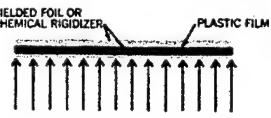
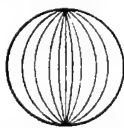


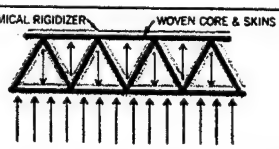
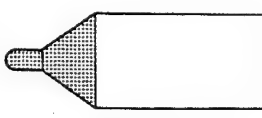
TECHNIQUE	TYPICAL APPLICATION	
VARIABLE GEOMETRY (RIGID COMPONENTS)	 HINGE TELESCOPE	 PANELS & SENSORS
VARIABLE GEOMETRY (ELASTIC RECOVERY)	 SPRING FLEXIBLE PLASTIC FOAM	 PASSIVE COMMUNICATIONS SATELLITE CREW TRANSFER TUNNEL
INFLATABLE BALLOON	 OPTIONAL METALLIZING PLASTIC FILM OR FABRIC	 PARAGLIDER
AIRMATT	 WOVEN FABRIC & SEALANT DROP THREADS	 LIFTING-BODY REENTRY VEHICLE
RIGIDIZED MEMBRANE	 YIELDED FOIL OR CHEMICAL RIGIDIZER PLASTIC FILM	 ECHO 2
FOAMED-IN-PLACE	 RIGID PLASTIC FOAM PLASTIC FILM OR FABRIC	 SOLAR COLLECTOR
EXPANDABLE HONEYCOMB	 CHEMICAL RIGIDIZER WOVEN CORE & SKINS	 EXPANDABLE MOL

FIGURE 36.—Expandable structure deployment technique and applications.  
(Courtesy of Space/Aeronautics.)

### ***Inflatable Structures***

"Inflatable structures" are defined as fabric or film envelopes that maintain structural integrity by internal pressurization. Generally they are spherical, cylindrical with dome ends, or toroidal shells. An example of this concept is the inflato plane (designed by Goodyear Aircraft Co. for the U.S. Army), an aircraft with a 28-ft wingspan, that weighs only 290 lb. A variation of the principle is found in the Airmat (also developed by Goodyear), in which varying lengths of integrally woven drop threads create cross-sectional shapes, such as flat panels or airfoils. (See fig. 37 (a), (b), and (c) from ref. 41.) Potential space applications include extensions for space stations, space furniture, and rendezvous docks.

### ***Chemically Rigidified Structures***

Deployed structures given rigidity through a chemical reaction are made by impregnating a film fabric with a rigidifying resin. Woven, fluted, or corrugated sandwiches also may have application in these structures to provide buckling resistance under bending loads. Three basic types of chemical rigidification systems are: (1) plasticizer boil-off, (2) gas catalyst, and (3) radiation-cured systems.

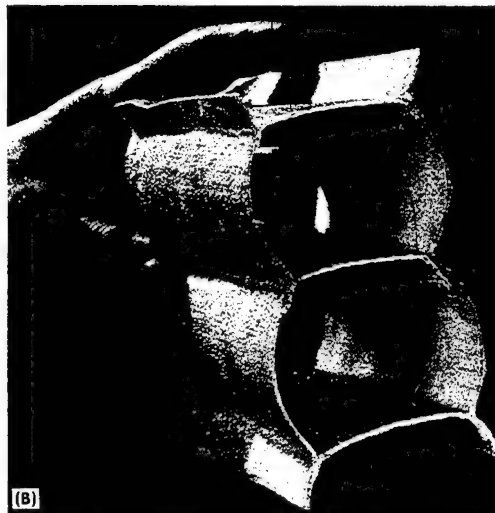
The best plasticizer boiloff system utilizes gelatin and water as a plasticizer. Gelatin rigidification systems were developed by the U.S. Air Force, Swift & Co., Monsanto Chemical Co., and Hughes Aircraft Co. for the Air Force. Such a system permits a structure to be deployed, rigidified, and tested on Earth, then plasticized and packaged for launch. Gelatin can be replasticized repeatedly by exposure to humidity. In the space environment, vacuum causes water migration out of the structure and produces rigidity.

Gas-catalyzed urethanes, developed by Archer Daniels Midland Corp. for the U.S. Air Force, are cured into rigid plastic upon exposure to water vapor. Another example, developed by National Cash Register Co. for the Air Force, is a fast-reacting system that utilizes a vinyl monomer and an amine catalyst. The catalyst, locally introduced by a fine spray or gas, will automatically cure and propagate throughout the entire structure. Both of these gas-catalyst systems require refrigeration to extend storage life.

Radiation-cured systems employing epoxy and polyester resins have been developed by Hughes Aircraft Co. for the Air Force and NASA. These systems require exposure to solar radiation for initiation and/or continuation of the rigidification process. A foam-in-place rigidification system was once considered for space use, but tests of several versions for the Air Force and NASA showed it to be impractical because of its complexities and poor strength-to-weight ratios.



FIGURE 37.—Three expandable structure sections: (A) segment of rigidified-foam solar reflector which would be activated in space; (B) rigidified honeycomb section; and (C) sample of airmat. (*Courtesy of Space/Aeronautics.*)



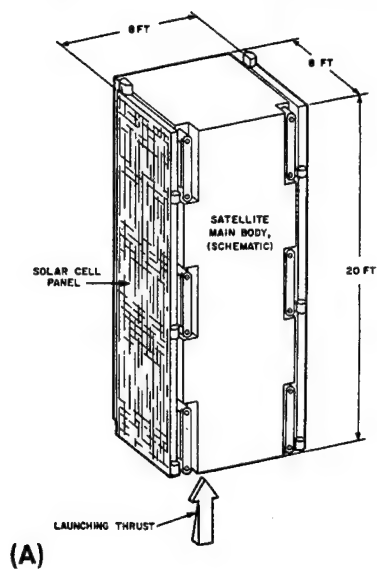
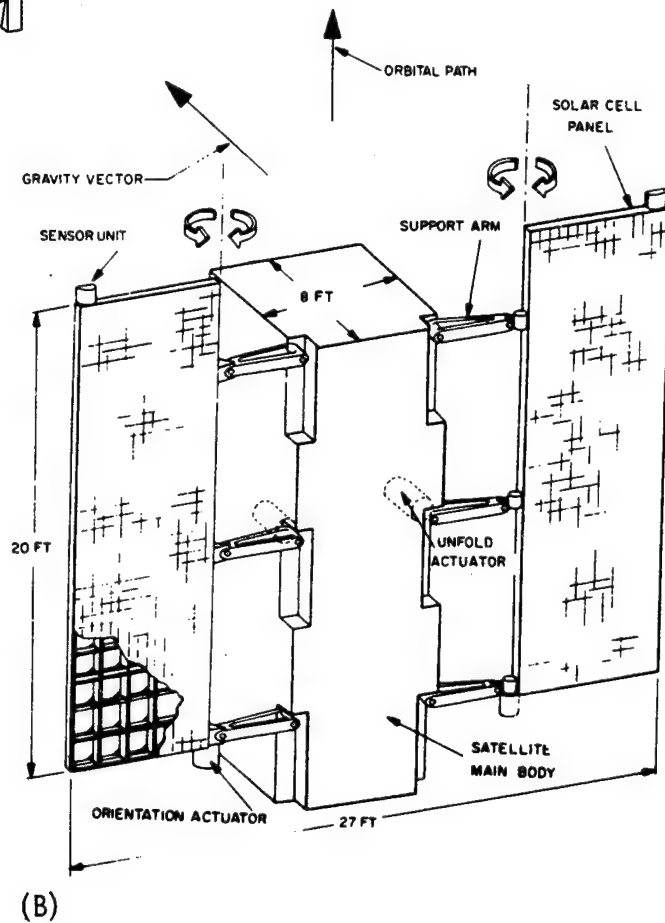


FIGURE 38.—Solar cell array: (A) stowed position for launching, and (B) operating position. (Courtesy of Radio Corporation of America.)



**Unfurlable Structures**

Unfurlable structures are defined as bodies that are mechanically deployed by hinges, sliding sections, and telescoping members. Conventional materials often can be designed to fold mechanically into a component package. This concept and its variations have been widely used by aerospace designers because conventional mechanical design procedures can be applied with high reliability. Figures 38 (a) and (b) and 39 (from ref. 44) are designs used for solar cells.

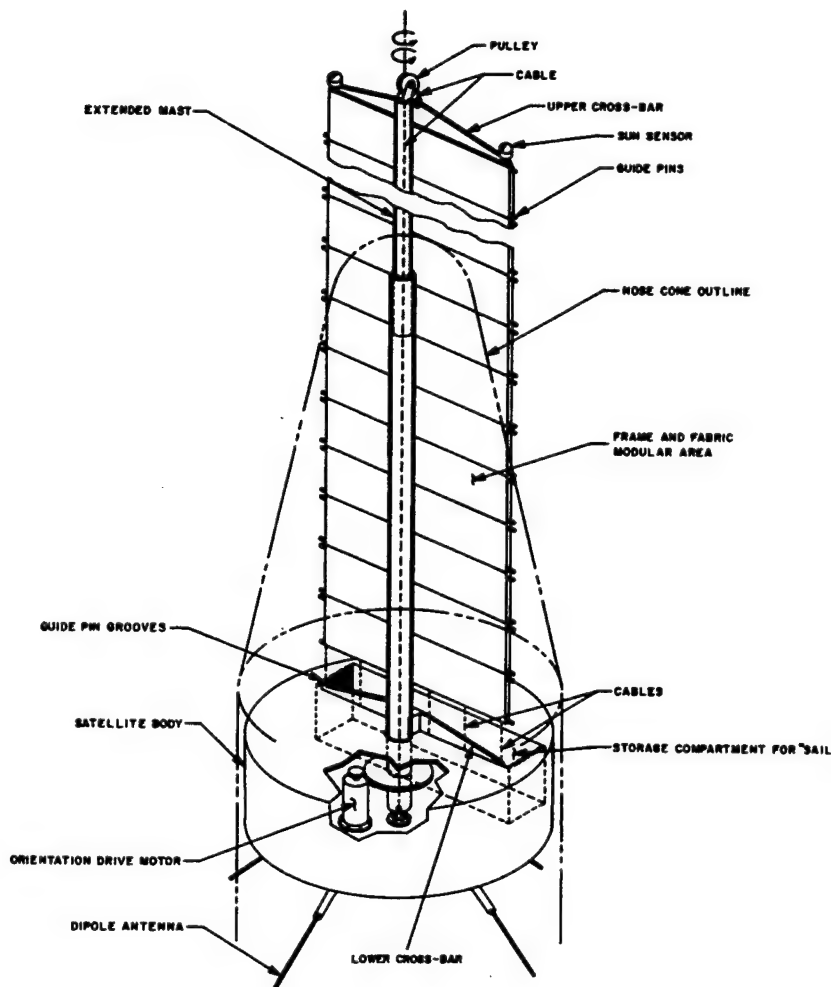


FIGURE 39.—Unfurlable sail type of solar collector. (Courtesy of Radio Corporation of America.)



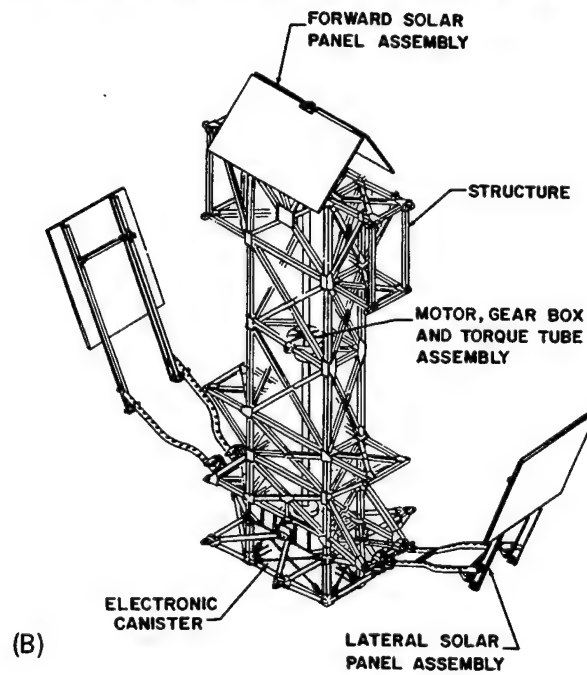
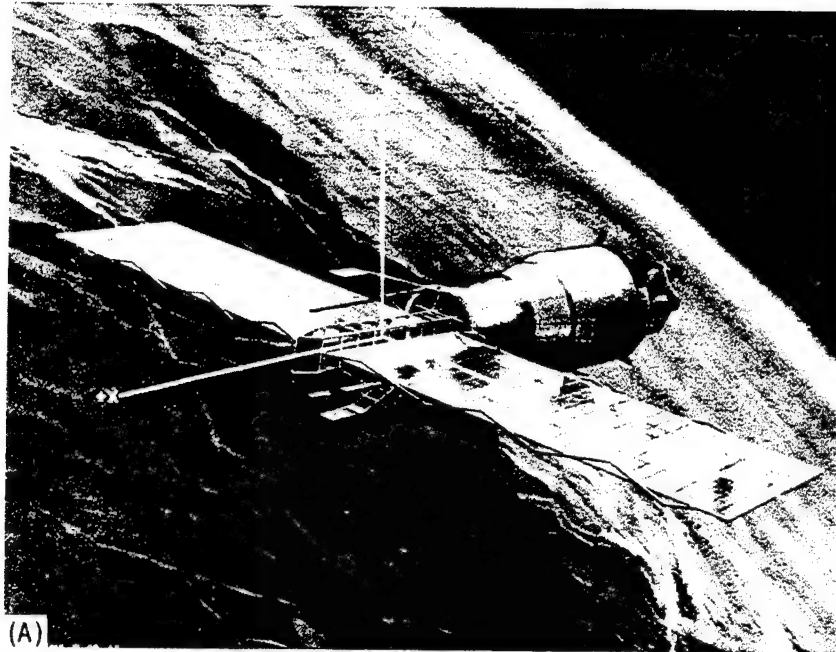


FIGURE 40.—Pegasus satellite and its center section: (A) showing orientation of axes, and (B) center section.

The Pegasus Satellite (Fig. 40 (a) and (b) from ref. 45), formerly called the Micrometeoroid Measurement Capsule, was an excellent example of this type of structure. A modular winged satellite launched for meteoroid detection in near-Earth orbits, it was developed by Fairchild Hiller Corp. for NASA under the supervision of Marshall Space Flight Center. Deployment of the detection panels was accomplished by a mechanical unfolding stress structure, giving a wingspan of 96 ft.

### ***Elastic Recovery Structures***

Both the Air Force and NASA have launched programs to develop expandable structures for aerospace applications, such as solar collectors (Geophysics Corp. of America, Viron Division; National Cash Register Co.; Goodyear Aerospace Corp.); reentry applications (Goodyear Aerospace Corp.); space stations (Goodyear Aerospace Corp. for the Air Force); expandable airlocks, crew tunnels, and hangars (Goodyear Aerospace Corp., Geophysics Corp. of America, Viron Division, for the Air Force and Narmco Division of Whittaker Corp. for NASA); and lunar shelters (Goodyear Aerospace Corp. for NASA). A recent report by Brink et al. (ref. 42) contains a detailed discussion of the development, feasibility, and applicability of the elastic recovery concept to expandable structures.

These structures are defined as those that utilize the basic elastic properties of the materials to deploy and provide limited structural rigidity. Such a structure is packaged by compressing and folding it into an extremely small container. Upon release from the container, the stored potential energy of the material is sufficient to expand and rigidify the structure.

The STEM boom of De Havilland Aircraft Co. is an excellent example of this concept. Circular in cross section, the boom is slit along its entire length so that it can be rolled up on a drum that flattens the cross section. Upon extension of the boom, the flat ribbon springs back into its original circular cross section.

Figure 41 (from ref. 41) shows properties and applications of the expandable-structure techniques, and figure 42 (from ref. 41) compares structural-merit to packaged-to-deployed volume ratio.

## **BUILDING CONSTRUCTION APPLICATIONS**

### ***Pneumatic Structures***

Experience with novel structures in space may facilitate the use of such new ideas on Earth. Within the atmosphere, an inflatable structure is commonly referred to as a pneumatic structure; this has become an important concept for commercial structures. Membranes pre-

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PROPERTIES							
Structural Merit—Tension							
Structural Merit—Compression							
Structural Merit—Bending							
Expanded-to-Packaged Volume Ratio							
Reliability—Deployment (including curing & rigidization)							
Expected Reliability—Postdeployment							
Micrometeoroid Resistance							
Ability to Maintain Shape After Puncture							
Heat Resistance							
Shelf Life							
Contour Accuracy							
Configuration Choice Flexibility							
APPLICATIONS							
Shelters (portable)							
Shelters (non-portable)							
Reentry Vehicles							
Decoys							
Space Stations							
Rendezvous Dock							
Crew Transfer Tunnel							
Passive Comsats							
Solar Collectors							
Antennas							
Space Furniture							

FIGURE 41.—Properties and uses of expandable structures. (Courtesy of Space/Aeronautics.)

stressed by internal pressure completely enclose a volume or a number of separate volumes. The membranes are very thin stressed skins, generally built from sheet metal, fabrics, or fiber-reinforced plastics. They are so thin that, for all practical purposes, they cannot resist compression, bending, or shear, but only tension. Figure 43 (from

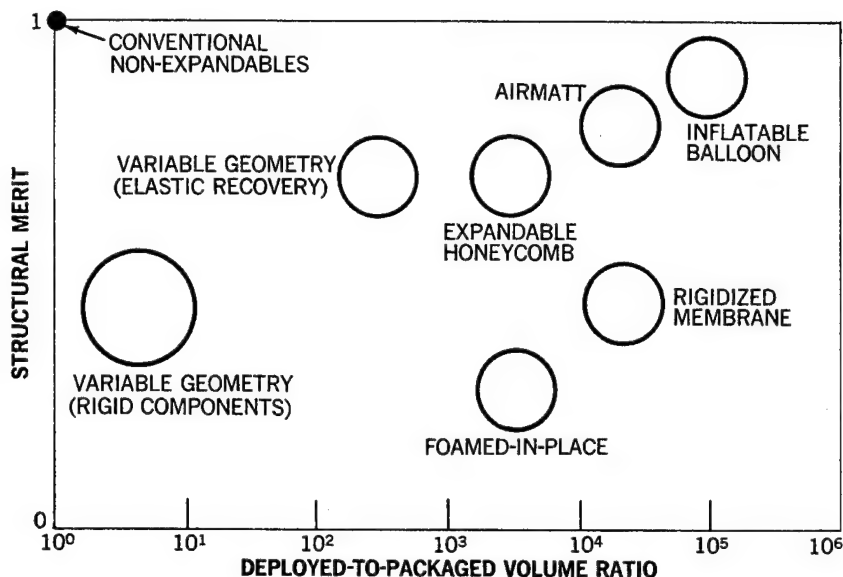


FIGURE 42.—Comparison of structural merit to deployed-to-packaged volume ratio of expandable structure techniques. (Courtesy of Space/Aeronautics.)

ref. 46) illustrates an advanced application of membrane structures called a "wavehall."

Structural membranes may be classified as either anticlastic or synclastic, depending on the curvature of the surface. "Anticlastic" means having opposite curvatures; i.e., having the center of principal radii at different sides of the observed tangent plane. A soap bubble and certain pneumatic structures are synclastic surfaces. The rubber raft is an excellent commercial application of this principle.

Concepts of an optimal structural form for two-dimensional components have long been in common use for the arch and suspension-cable structures. The extension of these concepts to curved surfaces or three-dimensional structures can be characterized by the structural membrane or minimum structure. Otto and Trostel have developed "sail-shells," stiffened pneumatic surfaces, and other tensile-stressed forms (ref. 46), which are based on the premise that structural form is determined by the equilibrium of forces rather than the geometry. In certain applications, a reversal of the tensile-stressed structure seems to result in a parallel reversal of stresses from tension to compression.

Pneumatic-structural systems, proposed by Lanchester in 1917, have made practical great large-span domes and hydraulic structures such as large-span dams. Figure 44(a) (from ref. 47) illustrates a pneu-

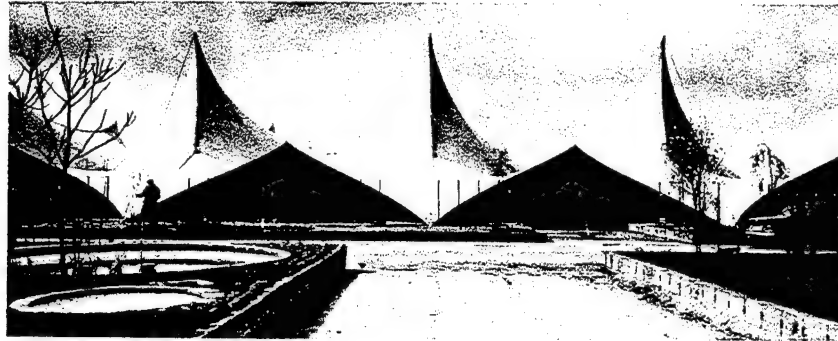
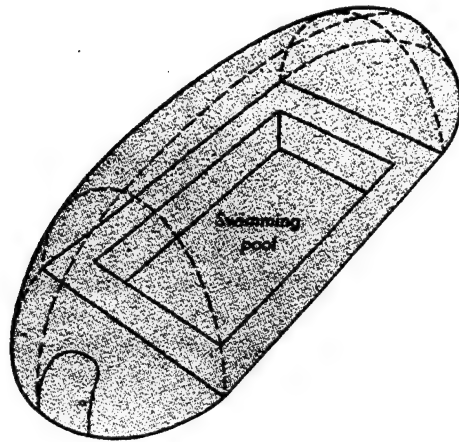


FIGURE 43.—A wavehall membrane structure.

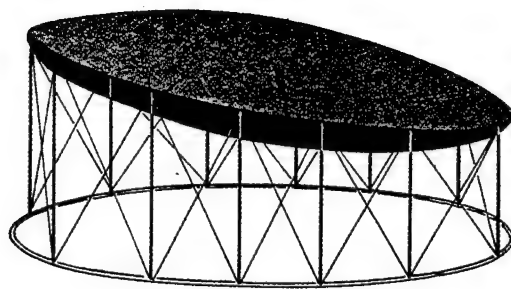
matic-roof application with thin plastic membranes inflated by a small pressure to create stable domes for swimming pools or other installations. An overpressure of only one-tenth or two-tenths of a psi is sufficient to hold up such a structure. Regular doors can be used, since, even if they are opened frequently, the loss of pressure in the large enclosed volume is negligible. Such losses, of course, are replaced intermittently under control of a pressure gage.

Koch and Weidlinger designed a balloon roof (fig. 44(b)) for a summer theater in the form of a lens inflated by a pump to an overpressure of 10 psf. And some 24 years ago, Wallace Neff began to produce stiff shells called "igloo houses" by pouring concrete over rubber balloons (fig. 44(c)). An inflated balloon supports a reinforcing steel mesh which is sprayed with a 1-in. layer of concrete from a concrete gun. After the concrete has hardened, the form is deflated and pulled out of the house through the door opening. The igloo house was invented by Neff and designed by Elliott Noyes and Mario Salvadori.

In 1962, in Essen, Germany, a pneumatically stressed balloon skin was sprayed with plastic from the inside to produce a weatherproof, insulated shell. The finished wall was a 20-mm-thick sandwich construction produced of glass fiber, polyester, and Perlite. After the polyester had set, the inside pressure (55 mm H<sub>2</sub>O) was lowered and openings were cut into the shell. In this construction a translucent skylight can be made. Another example in figure 45 (from ref. 48) illustrates a step-by-step version of this same technique. In this case a plastic film was inflated and held taut by compressed air while a 2-in. coat of urethane foam was sprayed onto the inside of the film. After drying, the shell was covered with metal reinforcing bars and openings were cut into the shell. The outer surface was then sprayed with 3 in. of concrete to make a durable, well-insulated, contemporary design.



(a)



(b)



(c)

FIGURE 44.—Modern enclosures: (a) pneumatic roof, (b) balloon roof, and (c) igloo (balloon) house.

### *Lattice Shells and Domes*

Lattice shells can have very high efficiency, use little material, and are stiff against buckling. Although their forms look complicated, they are very similar to their natural counterparts. (See fig. 46 from ref. 46.)

In figure 47 (from ref. 46), a method of forming a continuous lattice is shown. On a formwork (*A*), a steel wire fabric (*B*) and a second fabric (*C*) are placed in layers connected with wire ties (*D*). Between the ties are placed rubber balloons (*E*), which are blown up to pre-stress the wire fabric. After the balloons are deflated, only small voids remain; these are filled and stiffened with cement, concrete, or resins to form a continuous lattice shell. Finally, the rubber balloons are removed.

Shell systems may also be constructed from thin laths; in these systems a flat square grid is deformed into a spatially curved surface. Some examples are shown in figures 48 and 49 (from ref. 46). Such a shell may be erected over any arbitrarily chosen planform, such as one- or two-sided curvatures in dome or saddle form. Elastic, thin, flexible profiles of wood, aluminum, or steel are best suited as building materials for this adaptable method. Lattice shells can be easily erected and dismantled, and their form can be changed without destroying the structure.

Space-frame domes permit the spanning of large distances with relatively less material than required by other methods. In the geodesic dome designed by Fuller (ref. 49), the triangle and the pentagon are used in subdivision of bars of equal length (fig. 50). It is called "geodesic" because the vertexes of the curved figures that form its structure mark the arcs of great circles, known in geometry as geodesics. Radar domes built this way have withstood winds up to 150 mph and temperatures ranging from far below zero to over 150° in direct sun. The U.S. Pavilion at Canada's Expo '67 is an example of the geodesic dome.

The technology developed for space-frame domes with fiber-reinforced plastic skins can be interpolated into a folded-plate design for use in flexible and demountable dome structures. Although used mainly as roof structures, these plates may also be used as vertical walls to resist both vertical and horizontal loads. Combinations of folded-plate roofs and walls have been used to enclose large spaces. These plates may be ribbed, curved, or sandwiched for strength and rigidity (fig. 51).

Figure 52 is a proposed design employing the shatterproof sheet-panel concept. These panels are now available in a wide variety of sizes, colors, and light-transmission properties.

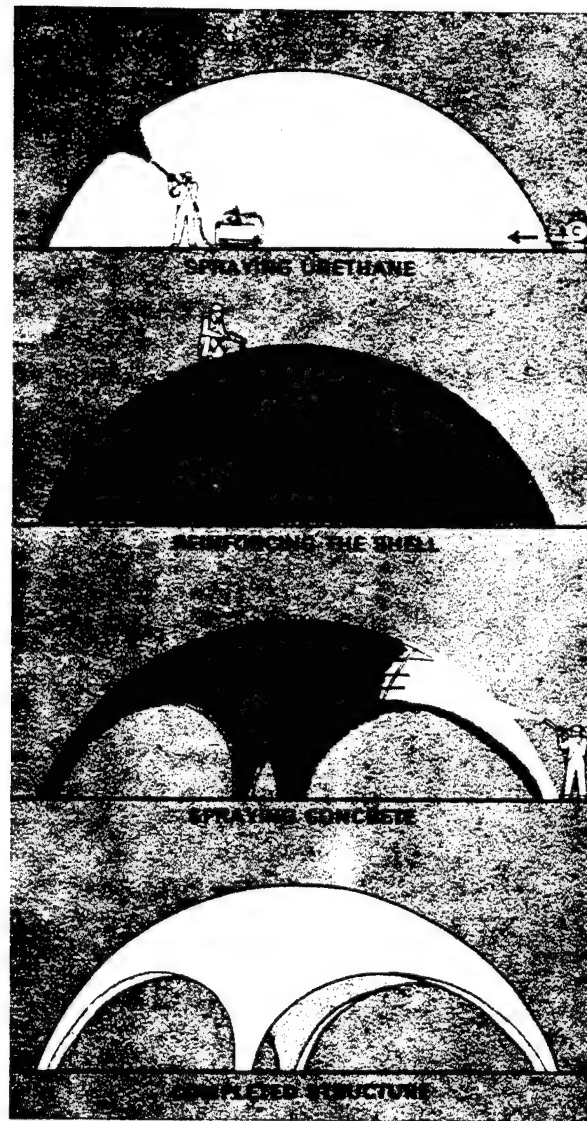


FIGURE 45.—Plastic building shell.

Figure 53 illustrates the formed-skin concept. A pavilion, constructed in 1950 for the U.S. Exhibition in Moscow, had a roof composed of a 20-ft-high, umbrellalike cluster of 16-ft-diameter hexagonal canopies supported on hollow columns. For translucency, as well as strength in shape, glass-fiber, mat-reinforced plastic was used as forming skin. Thickness ranged from  $\frac{1}{8}$  to  $\frac{1}{4}$  in., depending on the



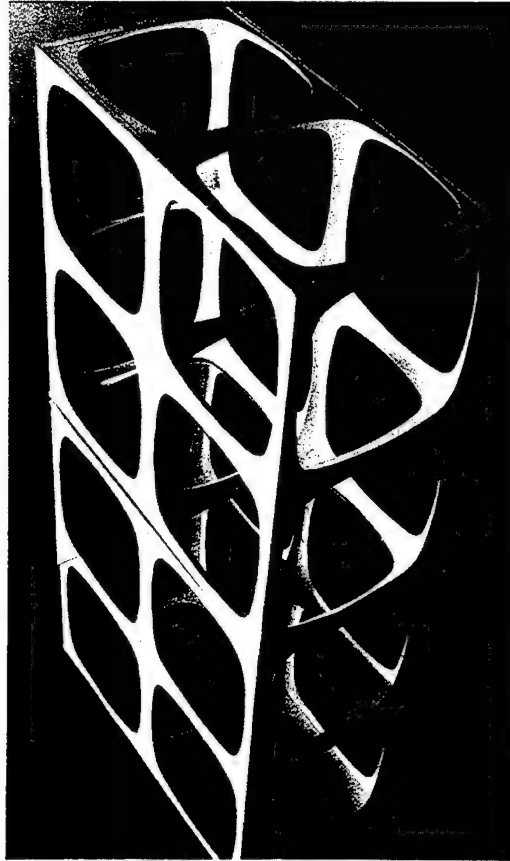


FIGURE 46.—Two-layered lattice shell made of sheet elements.

number of layers of material. The formed-skin concept, utilizing compound curvatures and shapes, is almost unlimited as a design approach.

Large spherical domes over radar installations (radomes), built on the principle of thin plastic membranes, do not interfere with the reception and transmission of electromagnetic beams. One recent type of dome structure for a ground-based radome uses foamed plastics which have very low electrical-loss characteristics. The edges of foam panels can be joined together with foam plastic or adhesive bonding to provide a uniform wall radome with a minimal effect upon transmission losses. This design may have wide uses, particularly for higher frequency applications.

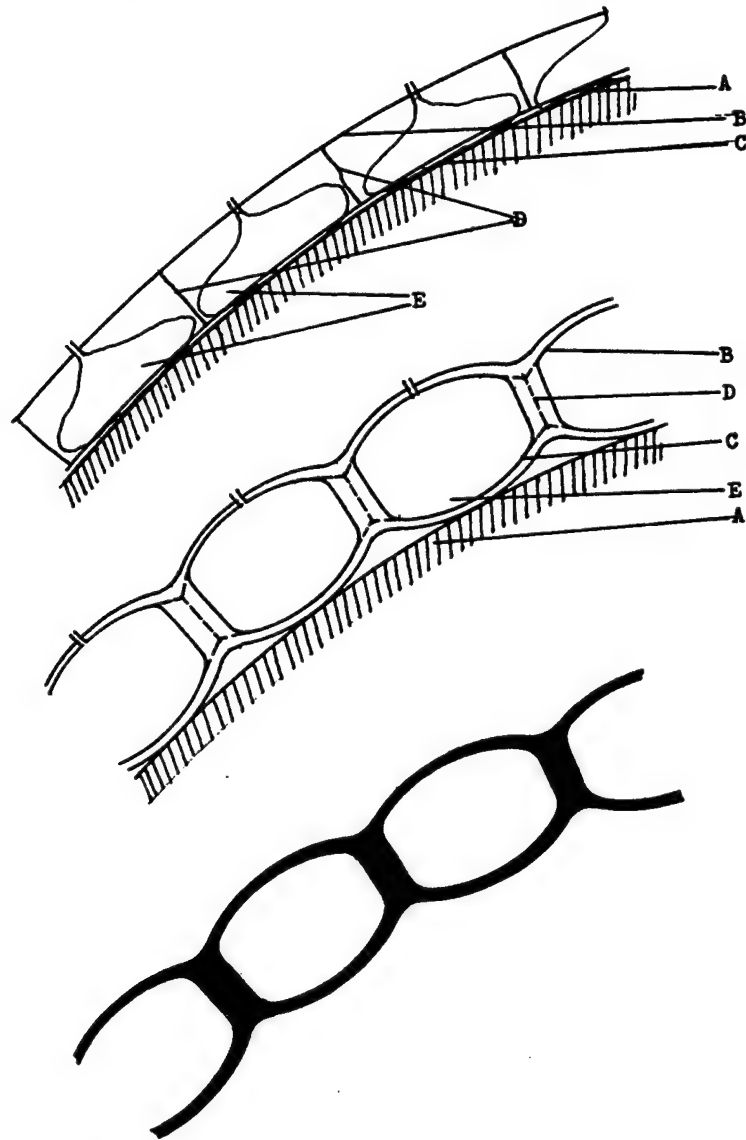


FIGURE 47.—Method of constructing a space structure.

The "House of the Future" at Disneyland uses self-supporting shells of fiber-glass-reinforced plastic in a unitized design that permits both brightness and stiffness. The 8- by 16-ft cantilevered, hollow-structure monocoque shells comprising the structure are glass-fiber

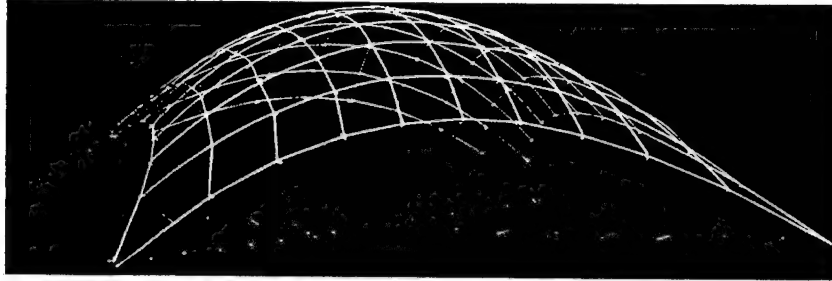


FIGURE 48.—Chain model.

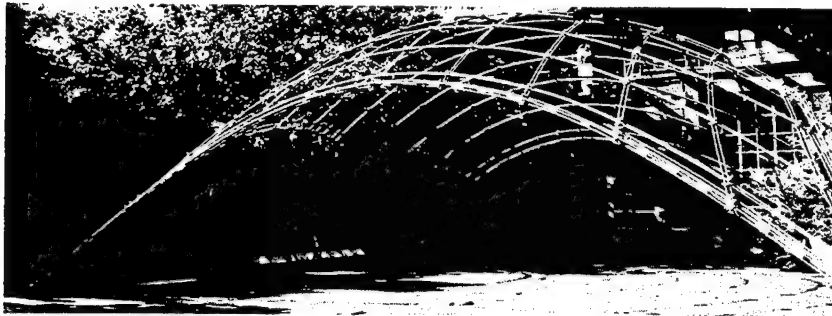


FIGURE 49.—Finished dome.

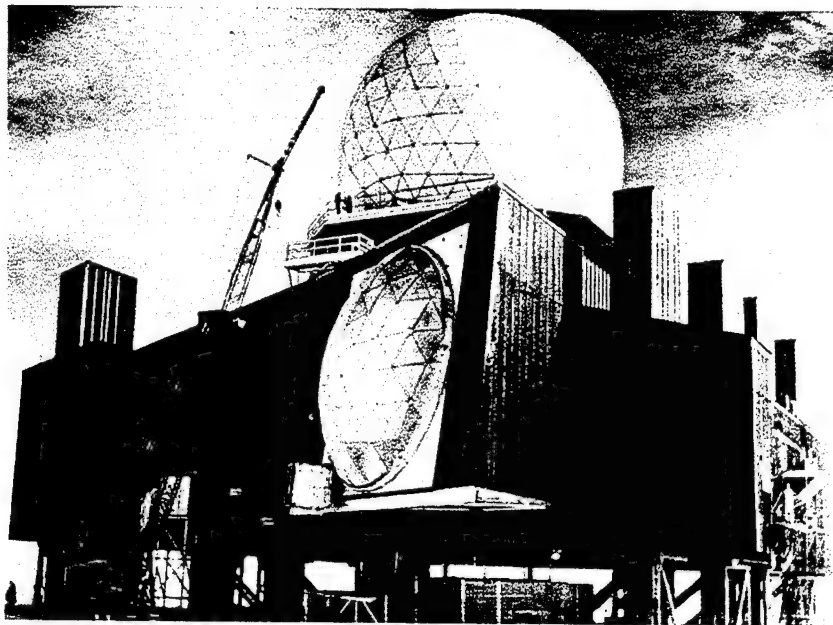


FIGURE 50.—Space-frame dome structures. (Courtesy of Owens-Corning Fiberglas Corp.)

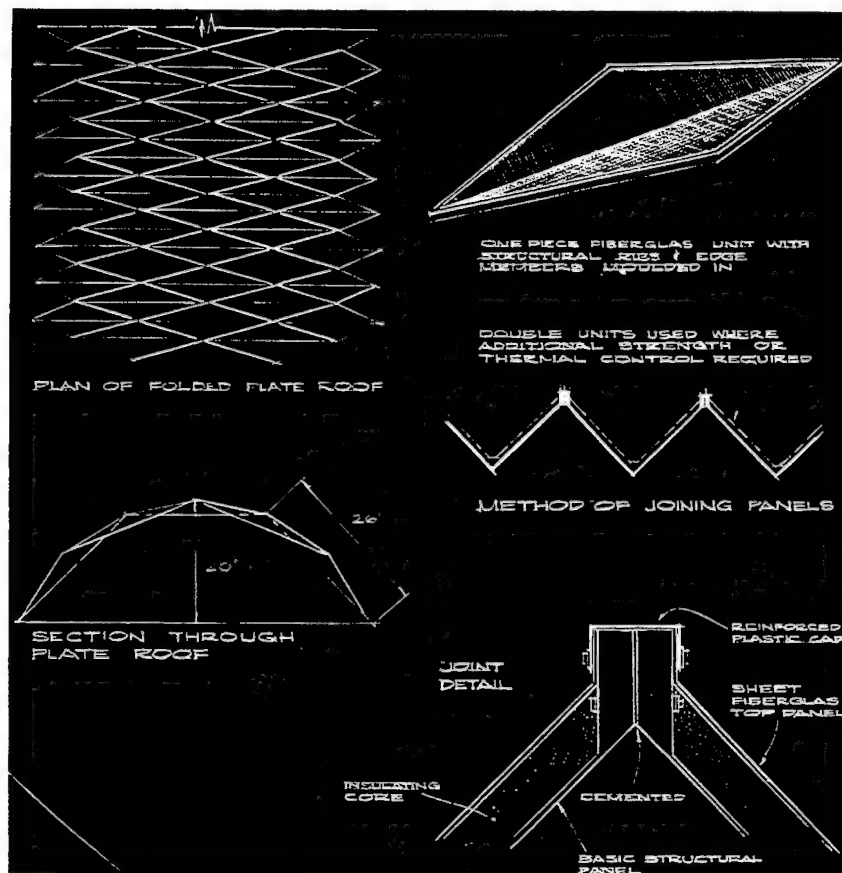


FIGURE 51.—Folded-plate design. (Courtesy of Owens-Corning Fiberglas Corp.)

roving and resin 0.3 in. thick. Another unitized design, shown in figure 54, is for portable shell units for motels.

"Buildings of the Future" include a home design with the basic roof and floor structures consisting of quarter moldings of fiber-reinforced plastic bonded together to form modules of shell construction; flat fiber-reinforced plastic panels and glass provide the other surfaces. In addition, a swimming pool, diving board, sun lounge, garden furniture, and woven fencing are molded in fiber-reinforced plastic.

A dynamic new design for tall buildings made of lightweight steel (or aluminum) girders supporting prefabricated, fiber-reinforced plastic modules molded on the shell principle has also been published (ref. 50). Another imaginative design of the future is a school building

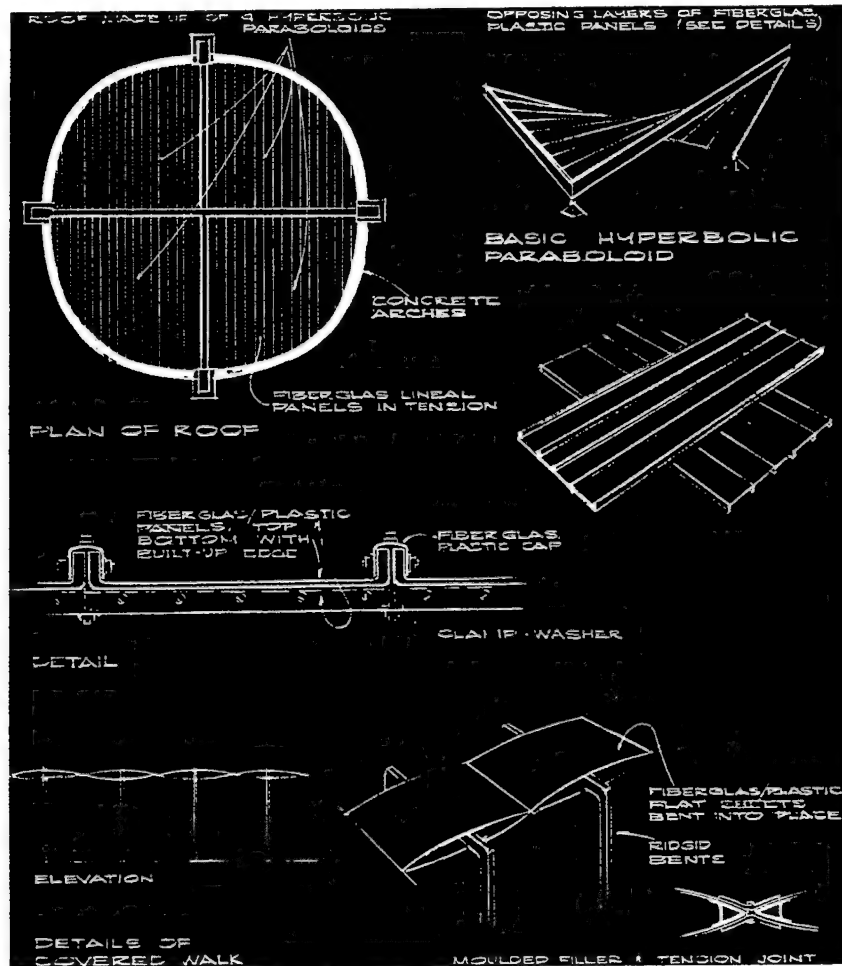


FIGURE 52.—Sheet-panel structure. (Courtesy of Owens-Corning Fiberglas Corp.)

with roof sections, supported on a stressed framework of fiber-reinforced plastic, forming a light, immensely weatherproof structure. Panels can vary from translucent to opaque, whereas the walls are clad in a sandwich construction of pigmented fiber-reinforced plastic with a core of insulating material.

Some other treatments of futuristic houses are filament-wound glass houses (shown in fig. 55 from ref. 51), which could be produced in round, conical, doughnut, or mushroom shapes by using filament-winding techniques. Double-wall construction, decorated with permanent, nonfading color, would provide insulation, and walls would

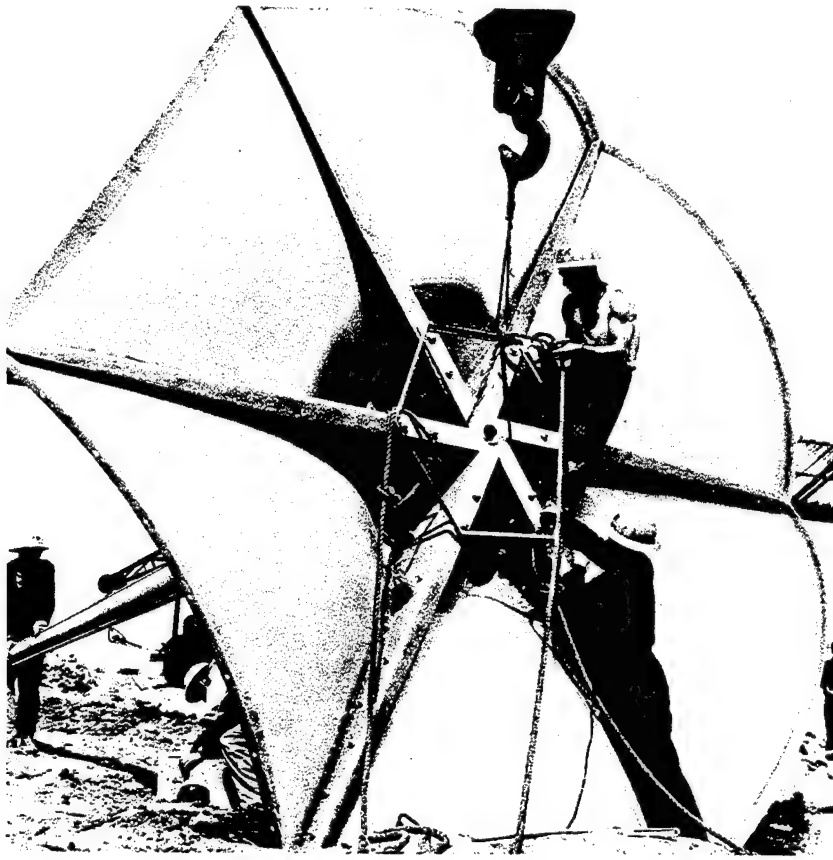


FIGURE 53.—Formed-skin structure.

be translucent to eliminate dark corners during the day. The entire structure could also be fabricated on the building site, once winding machines capable of such large tasks are developed.

An innovation not yet seen in this country is the isostatic-ribbed plate. As the reader knows, a plate is capable of developing grid action in any direction. Furthermore, any point in the plate may be considered as the intersection of two beams of a rectangular grid system, and any number of rectangular grid systems may be considered to be passing through a plate point. At each point there are two directions for which the bending stresses are, respectively, maximum and minimum, and for which the shear stresses are zero. If one indicates the principal stress directions at various points by crosses, principal stress-line patterns, or isostatics, may be plotted; these

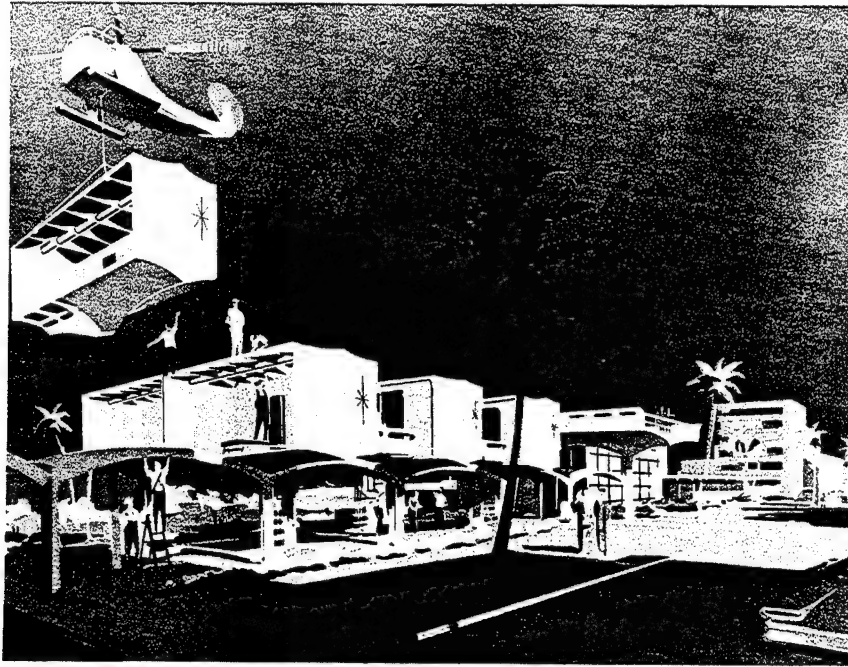


FIGURE 54.—Fiber-composite structures. (Courtesy of Owens-Corning Fiberglas Corp.)

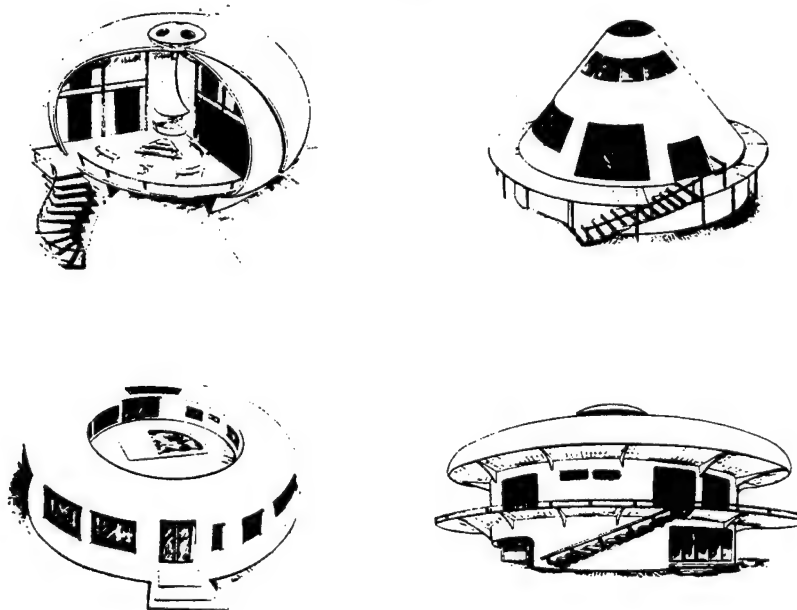


FIGURE 55.—Futuristic houses spun by large filament-winding machine.

represent the flow of stress in the plate. The pattern, of course, depends on the support conditions at the plate boundary and the loading. Since the shear stress is zero along isostatics, the plate may be visualized as a grid of curved beams that intersect at right angles and do not transmit loads to adjoining beams by shear action.

Isostatic lines form very interesting patterns and have been used by Nervi in floor designs (see frontispiece). The appearance of a ribbed plate can be made very attractive by constructing the ribs along isostatics.

These are examples of building ideas that aerospace research may help bring into wider use.



## Design Synthesis and Optimization

Design is the process of evolving a configuration to perform specific functions. It proceeds from the abstract to the concrete, and the initial concept is a relationship among ideas or geometrical forms. A final engineering design results from a series of problem-solving steps by which the configuration is evolved. This sequence of operations, called the "design process," carries a problem through analysis, synthesis, and evaluation and decision into optimization, revision, and implementation (ref. 52). In most engineering situations, the design process involves a number of successive iterations rather than a single direct solution of a closed-form equation that expresses the primary design problem (ref. 53). In general, the designer selects a configuration, analyzes it, selects another, analyzes it, selects a third, and so on; he also sets cost schedules and evaluates performance implications of alternate structures. In other words, the designer inputs the design requirements and makes systematic variations in concept, material, and detail to arrive at a set of designs that satisfy the design criteria and performance requirements.

To see the difference between the analysis and the design of a structure, suppose that the total geometry and load conditions are known and the material is given in the analysis. Then it is necessary only to analyze for structural behavior under the given load conditions. In the design process, however, only performance requirements are given; the geometry is not generally known. Thus the designer must generate a structure to meet specified requirements.

Synthesis is the fitting together of parts or separate concepts to produce an integrated whole. Because of the complexity of the process, constant revisions and reevaluations of the revised results often must be made and newly developed information added to the design considerations. Although the synthesis step formally begins after the design problem is well understood, some hypothesis for possible solutions will probably be suggested during earlier steps.

When the major design parameters must be set at specific values, optimization is used to find the best combination of parameter values to satisfy design requirements. This may be done mathematically, in which case all considerations associated with selection of the prime

solutions are set forth in an equation called the "criterion (merit) function."

The mathematical description of a design problem involves input (independent) and output (dependent) variables associated through transforming mechanisms analytically expressed. If all the variables and design parameters are known, one can calculate the criterion value that will give a measure of the excellence of a particular design. In general, we must compare a particular design choice with other possible choices to determine its excellence.

While a complete exploration of all physically realizable design parameters could be made, and the best set determined by elimination, the usual design situations have two constraints: functional and regional. Functional constraints essentially constitute the mathematical description of the archetype of the proposed design. Regional constraints set the allowable limits on design parameters, or on derived groups of parameters, representing more complex attributes of the proposed object. Thus, in the general optimization problem, three factors must be considered: (1) the criterion function, which is brought to a maximum or minimum, depending on which corresponds to an optimum through proper choices of design parameters; (2) the functional constraints; and (3) the regional constraints.

To state the problem mathematically (ref. 52), let us lump all of the variables together, so that the set  $(X_1, \dots, X_n)$  contains the design parameters, input variables, and output variables. The criterion, represented by  $U$ , takes on the values of the criterion function  $U(X_1, \dots, X_n)$ ; the set of functional constraints is represented by  $(\psi_1, \dots, \psi_m)$ ; and the regional constraints by  $(\phi_1, \dots, \phi_p)$ .

For the regional constraints, the  $i$ th one will be constrained between the lower limit  $l_i$  and the upper limit  $L_i$ .

Thus, the optimization problem is described by the following set of equations:

$$U = U(X_1, \dots, X_n) \rightarrow \text{optimum}$$

$$\psi_1 = \psi_1(X_1, \dots, X_n) = 0$$

$$\vdots$$

$$\vdots$$

$$\psi_m = \psi_m(X_1, \dots, X_n) = 0$$

$$l_1 \leq \phi_1(X_1, \dots, X_n) \leq L_1$$

$$\vdots$$

$$\vdots$$

$$l_p \leq \phi_p(X_1, \dots, X_n) \leq L_p$$

To give a geometrical interpretation of the analytical description, let us consider a three-dimensional space in which we can plot two variables  $(X_1, X_2)$ , since higher order spaces, or hyperspaces, are impossible to visualize. Let  $X_1$  correspond to the  $x$ -axis and  $X_2$  to the  $y$ -axis; the criterion value,  $U$ , corresponds to the  $z$ -axis. In order to plot  $n$  variables,  $X_n$ , an  $(n+1)$ -hyperspace is required. The  $X_1$ - and  $X_2$ -axes form a basis plane over which the criterion (merit) functions,  $U(X_1, X_2)$ , are constructed.

The functional constraint  $\psi_1(X_1, X_2)=0$  describes a curve on the basis plane. Projection of this curve onto the criterion surface gives a curve in space corresponding to  $\psi_1(X_1, X_2)=0$ ; while at the same time, the curve conforms to the surface  $U(X_1, X_2)$ . This space curve "rises" or "falls" depending on the shape of  $U$ . If the optimum corresponds to a maximum, the highest point on the curve is sought; if it corresponds to a minimum, the lowest point is sought. In either case the  $X_1, X_2$  coordinates of the optimum point will satisfy the constraint  $\psi_1$ , since the projection of its curve on the surface,  $U$ , was followed. If there is a second functional constraint,  $\psi_2(X_1, X_2)=0$ , the two curves corresponding to the two constraints will intersect at a particular point, or possibly at several points on the basis plane. In an  $n$ -dimensional problem,  $\psi_1$  and  $\psi_2$  will intersect to form a new hypercurve that will still be on the  $n$ -dimensional basis plane.

Now if we project the curve and the  $n$ -dimensional basis plane to the criterion function surface, a curve is traced on the latter surface. If we move along the projected curve, we shall rise or fall according to the shape of the criterion function surface. As before, the highest point on the curve is sought for a maximum; the lowest point, for a minimum.

Consider now the applications of regional constraints on a two-dimensional basis plane. The relation  $\phi_1(X_1, X_2)$  represents a family of curves on the  $X_1, X_2$  basis plane. Of this family, one extreme is  $\phi_1(X_1, X_2)=l_1$ , and the other is  $\phi_1(X_1, X_2)=L_1$ . Projection of the region between the two extreme curves to the criterion function surface defines a region on the surface that may be explored for an optimum. The introduction of a second regional constraint  $l_2 \leq \phi_2(X_1, X_2) \leq L_2$  leads (upon projection) to another region on the criterion function surface that may be explored for an optimum. The two curved strips of area will intersect on the basis plane to form a four-sided area, each of the sides being a segment of one of the limiting curves. Projection of this area onto the criterion function surface maps a specific region in which the optimum can be found. If other regional constraints are added, a region of many sides (equal to the number of constraints) must be projected onto the criterion function surface.

### STRUCTURAL DESIGN-SCIENCES APPROACH

Although stringent demands may be placed on materials to achieve overall design efficiency, it is important to utilize materials effectively. Although many aspects are involved in the effective utilization of materials, particularly under severe environmental conditions, three basic factors are summarized in figure 56 (from ref. 54): design, structure, and materials. The optimum design of a system requires consideration of all three factors simultaneously.

Aircraft, spacecraft, surface ships, submarines, and other vehicles have configurations, overall loads, and leading dimensions specified within rather narrow limits by performance requirements (ref. 55). The structural designer has some freedom within the confines of the leading dimensions to subdivide the structure with suitable stiffening systems to achieve a minimum-weight design (see fig. 57), but he must select materials that meet the particular structure and design conditions.

The design-sciences approach synthesizes the statement of the

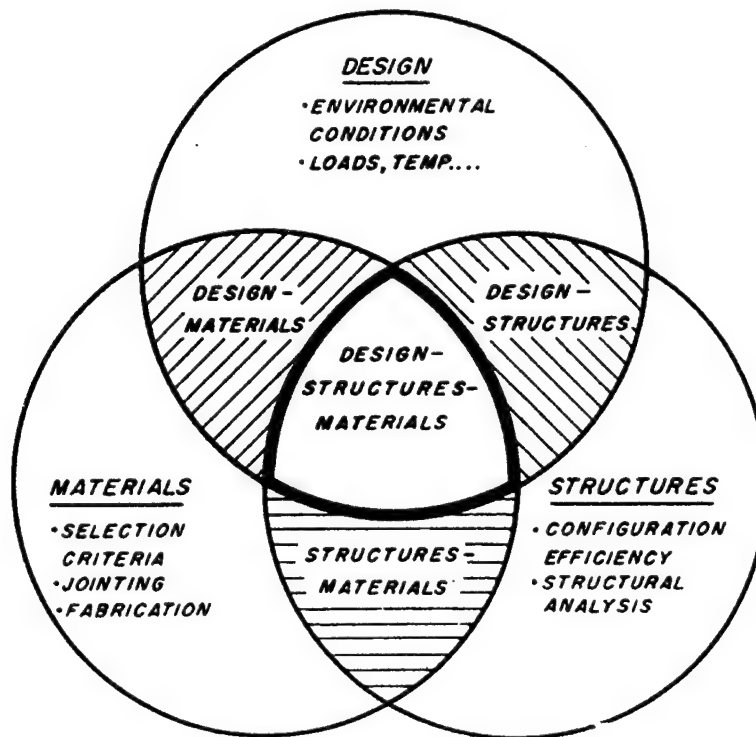
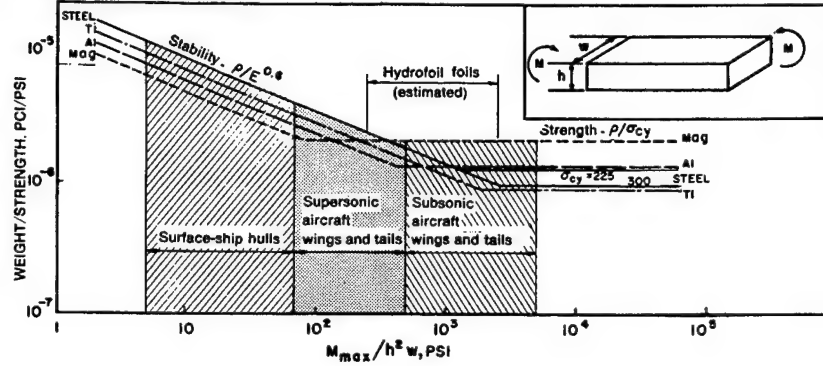
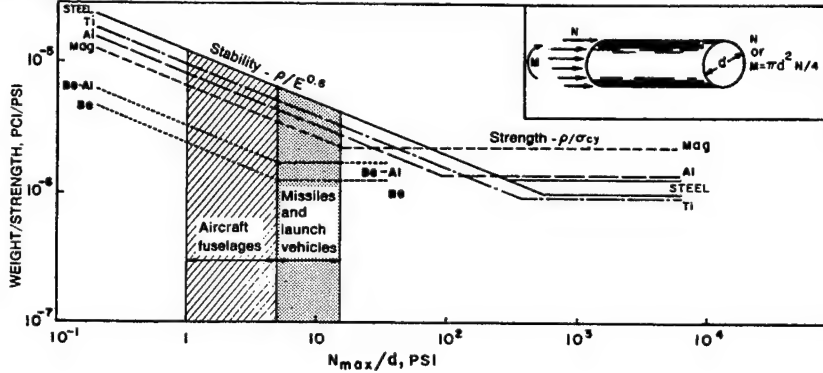


FIGURE 56.—Nature of the interplay.

## STIFFENED-BOX-BEAM APPLICATIONS



## STIFFENED-CYLINDER APPLICATIONS



## PRESSURE-VESSEL APPLICATIONS

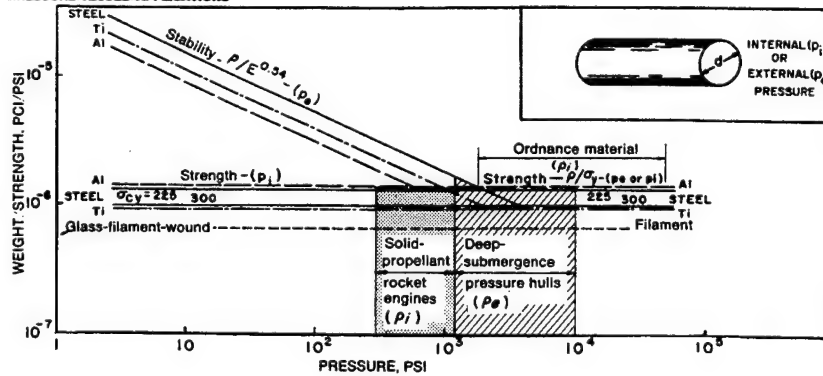


FIGURE 57.—Examples of design-sciences approach. (Reprinted from *Astronautics and Aeronautics*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

design problem by using certain design indices to combine the external loads and leading dimensions. It then uses idealized structural configurations, such as stiffened box beams and stiffened cylinders, to establish optimum designs from which to evaluate the comparative efficiencies of various materials.

This approach reached maturity within the last two decades. It permits the engineer to establish significant design parameters to evaluate the efficiency of various structural configurations and materials, using minimum weight as the criterion of optimum design.

As a broad generalization, the results of various types of minimum-weight analyses of representative structures can be expressed in the form:

$$W = SMD^m$$

where

$W$  = weight efficiency factor

$S$  = structural efficiency factor

$M$  = material efficiency factor

$D$  = structural design index

$m$  = exponent ( $0 < m \leq 1$ )

Although the weight efficiency factor,  $W$ , can be interpreted in several different ways, it is expressed in the form of a weight/strength ratio in the aerospace field. The structural efficiency factor,  $S$ , is generally a nondimensional quantity, whereas  $M$  is generally a density/modulus ratio representative of the material efficiency. The design conditions involving the external loads and leading dimensions are characterized by the structural design index,  $D$ . Thus, this equation, shown graphically in figure 57, represents the interrelationship among structures, materials, and design. Examples of the design-sciences approach follow:

#### **Box-Beam Structures**

Surface-ship hulls, aircraft wings and tails, hydrofoils, Army combat vehicles, and military bridges can be characterized in idealized form as stiffened box beams under bending loads. (See fig. 57 from ref. 55.)

Longitudinal stiffeners are I, Z, or hat sections, supported by transverse stiffeners that are transverse ribs at optimum spacing.

$$W = \frac{\text{weight}}{\text{strength}} = 2.38 \left( \frac{\rho}{E^{0.6}} \right) \left( \frac{M}{h^2 w} \right)^{-0.4}$$

stability limitation due to buckling

with

$$\begin{aligned}
 S &= 2.38 \\
 M &= \rho/E^{0.6} \\
 D &= M/h^2w \\
 m &= -0.4
 \end{aligned}$$

When the buckling strength equals or exceeds the compressive yield strength of the material,

$$\frac{\text{weight}}{\text{strength}} = \frac{\rho}{\sigma_{cy}}$$

Therefore,

$$M = \frac{\rho}{\sigma_{cy}}$$

when strength limitations govern, or

$$M = \frac{\rho}{E^{0.6}}$$

when stability limitations govern.

### ***Stiffened-Cylinder Structures***

Aircraft fuselages, missiles, and launch vehicles can be idealized as stiffened cylinders under bending and axial compression, respectively. (See fig. 57.)

The longitudinal stiffeners are I, Z, or hat sections, supported by transverse frames which are I, Z, or hat sections.

$$W = \frac{\text{weight}}{\text{strength}} = 1.25 \left( \frac{\rho}{E^{0.6}} \right) \left( \frac{N}{d} \right)^{-0.4}$$

with

$$\begin{aligned}
 S &= 1.25 \\
 M &= \rho/E^{0.6} \\
 D &= N/d \\
 m &= -0.4
 \end{aligned}$$

Also,

$$\frac{W}{S} = \frac{\rho}{\sigma_{cy}}$$

### ***Pressure Vessels***

Submarine pressure hulls, solid-propellant rocket motors, and various ordnance can be treated as pressure vessels under external or internal pressure.

For an I-, Z-, or hat-frame-stiffened cylinder ( $L/D=1$ ),

$$W = \frac{\text{weight}}{\text{strength}} = 1.5 \left( \frac{\rho}{E^{0.4}} \right) (p)^{-0.46} \quad \text{external pressure}$$

Also,  $W/S = \rho/\sigma_{cy}$ , the strength-limitation region, is valid for both internal and external pressure.

#### Further Development of the Approach

The foregoing examples demonstrate the application of the design-sciences approach to the evaluation and improvement of material properties in terms of their potential applications. This approach was also used to identify structural design limitations in the case of deep submersibles.

If we accept improved weight/strength efficiency as a desirable goal, we can summarize potential improvements, as shown in figure 58. Here, if a given design application has a design-index range corresponding to  $D_1$ , then design, structures, and materials (density and elastic modulus) improvements can lead to greater weight/strength efficiency. On the other hand, if the design-index range corresponds to  $D_2$ , only material improvements (density; yield or ultimate, strength; ductility) can contribute to weight/strength efficiency.

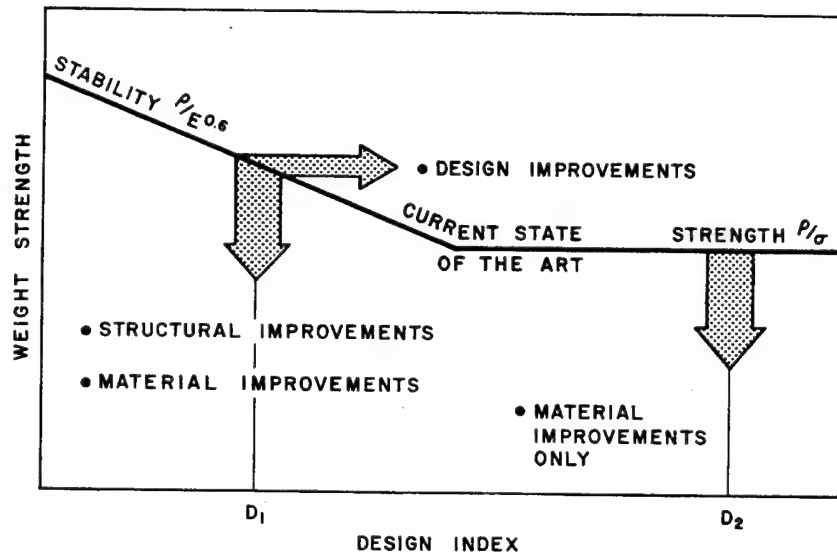


FIGURE 58.—Potential improvements in current state of the art. (Reprinted from *Astronautics and Aeronautics*.) (Courtesy of American Institute of Aeronautics and Astronautics.)



Although conclusions concerning improved materials depend on the design-index range corresponding to the application, the surprising fact is that the various applications indicated in figure 57 are characterized by rather narrow design-index ranges. This permits valid conclusions regarding current designs to be drawn from this approach and also permits rather safe conclusions for future designs. As a result, this approach can help in technical decisions for long-range material-development cycles.

The design-sciences approach is reasonably well developed in certain aspects and can be effectively employed in the following areas:

- (1) To evaluate current and experimental materials over a broad temperature range extending from cryogenic to elevated temperatures
- (2) To provide guidelines for identifying and developing improved material properties for projected applications

Further investigations in the following areas could greatly advance this approach (ref. 55):

- (1) Engineering studies of structures to provide design-index data for current and projected applications
- (2) Determinations of why applications fall within a narrow range of design-index values
- (3) Study of various design configurations to alleviate or remove the design limitations
- (4) A project relating minimum-weight results to cost for optimum structures

### FULLY STRESSED DESIGN

For a structure under multiple-loading conditions, the method of fully stressed design proportions the structural members by equalizing the allowable stress in any member in at least one loading condition (ref. 56). If analysis shows that a certain member is overstressed in a critical load condition, the method of fully stressed design increases the area of that member enough to remove the overstress. Conversely, this method does the opposite if the member is understressed.

For structures with so-called hybrid action, each member must be designed with consideration of its effect on other members. For this type of structure, the convergence is generally slow; and the resulting repetition of analysis and fully stressed redesign often tends to simplify the structure by eliminating some of its members.

The minimum-weight design of a structure is an arrangement of the structural element in which all the design requirements (such as stresses, deflections, and geometric constraints) are satisfied, while the total weight of the entire structure is minimized. This minimum-weight design can generally be set up as a mathematical programming problem. Efficiency of the fully stressed design and its relationship to

a minimum-weight design has been discussed by a few investigators. Although Schmidt (ref. 57) has argued that a minimum-weight design may be selected from among fully stressed designs, Schmit (ref. 58) has shown that a fully stressed design is not necessarily a minimum-weight design. Under some loading conditions, in fact, the fully stressed method may lead to an inefficient design. Razani (ref. 56) has sought to determine when a fully stressed design has minimum weight and when it has not; when it has not, he suggests a method of determining optimum structure.

The iterative, fully stressed design usually changes the configuration of the structure considerably in the initial cycles, but successive changes generally result in progressively fewer modifications.

In the method of fully stressed design, the problem of convergence is studied within the range where changes in area or stiffness of structural members are small. It is assumed, in addition, that the critical loading condition for each member does not change abruptly because of a small change in design configuration; thus, the critical forces in the members can be treated as continuous functions (ref. 56).

#### ***Relationship of Fully Stressed and Minimum-Weight Designs***

For determinate structures (see fig. 59) then, the fully stressed design is the minimum-weight design; whereas for indeterminate structures, the critical force in each member is not only a function of the applied loading but also a highly nonlinear function of the areas of all the members of the structure. Consequently, the fully stressed design is not always an optimum design.

#### **Condition of optimality**

$$\lambda = (I - B^T)^{-1} \rho L > 0 \quad \text{Kuhn-Tucker optimality conditions}$$

where

$\lambda$ 's = optimality coefficients

$B = m \times m$  design variation matrix,  $B = (b_{ij})$

$$b_{ij} = \left( \frac{1}{\sigma} \right) \left( \frac{\partial \bar{F}_i}{\partial A_j} \right) = \frac{\partial A_i}{\partial A_j}$$

$B^T$  = transpose of matrix  $B$

$I = m \times m$  unit matrix

$\rho$  = material density or unit weight of material

$L$  = length of section

$\bar{F}_i$  = critical load of  $i$ th member

$\sigma$  = corresponding stress for the critical load

$A_i$  = area of  $i$ th member

$m$  = number of members

When the fully stressed design is not optimum, the productivity test can be used to determine and separate the free variables from the fully stressed ones (refs. 56 and 59),

$$P_i = \frac{\partial V}{\partial A_i} \cong L_i + \sum_{\substack{j=1 \\ j \neq i}}^m (A_j^n - A_j^o), \quad i=1, 2, \dots, m$$

where

$P$  = productivity coefficients

$A_j^n$  = final area of  $j$ th member obtained by an iterative, fully stressed design while keeping the area of the  $i$ th member constant and equal to  $A_i^o + \Delta A_i$

$A_j^o$  = initial area of the  $j$ th member before change in the  $i$ th member

$\Delta V$  = total change in the volume of the truss due to a change  $\Delta A_i$  — the  $i$ th member

In this case, dimensionality of the problem is reduced and optimization is decentralized to an optimal search for free variables and to the fully stressed design of the remaining variables. In general, the faster

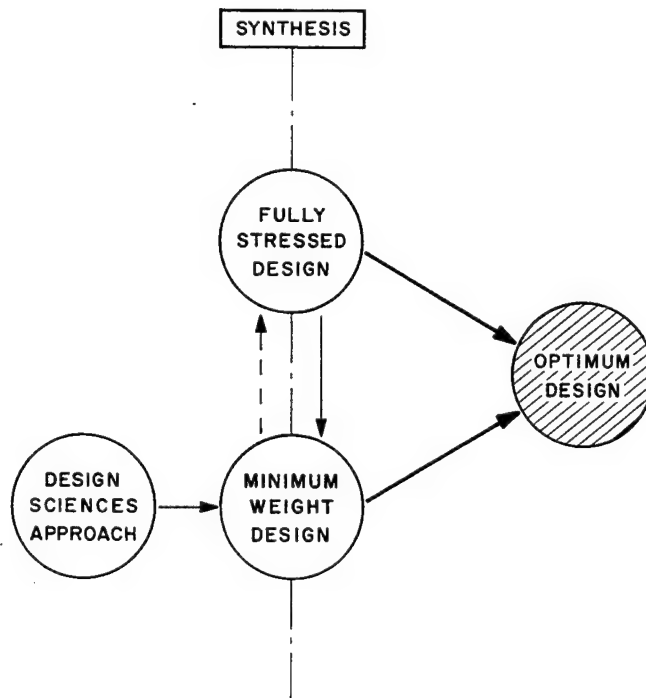


FIGURE 59.—Relationship of design steps.

the convergence rate of the iterative, fully stressed design, the more likely the optimality of the design. Consequently the fully stressed design of structures with normal action is more likely to be optimum. We, therefore, have another approach for structural-design optimization.

### STRUCTURAL OPTIMIZATION METHODS

We may divide the classical numerical optimization methods into three general groups: (1) "Perturbation Methods," which include the indirect methods of Adjoint Functions and Perturbation Functions; (2) the "Quasilinearization Methods," which also include the indirect methods of the "Generalized Newton-Raphson," a "Modified Generalized Newton-Raphson," and the "Modified Quasilinearization"; and (3) "Gradient Methods," which are direct methods including the "Method of Steepest Descent" and the "Modified Method of Steepest Descent." An excellent paper which analyzes and compares these conventional methods was recently given by Lewallen and Tapley (ref. 60). We shall briefly discuss one of the more recent methods devised for structural optimization, called the "Random-Sampling (RS) Method."

#### *Random-Sampling Method*

The general problem of optimization with arbitrary constraints is treated by means of random numbers and Monte Carlo sampling techniques. Kiciman (ref. 61) demonstrated the validity of the technique by comparing his results on structural synthesis problems with published results using the gradient method. Although the design configurations produced by this approximating technique are not better than those given by the gradient method, they are acceptably close. A specific application to the minimum-weight design of a Z-stiffened compression panel is also given and the results checked against values computed by the designer using conventional methods. Indications are that this generalized and readily applicable synthesis approach will enable the designer to investigate several different design concepts for their relative design values without waste of time and effort. Two main advantages of this technique are: (1) no restrictions are placed on any of the constraint and merit functions, and (2) any number of variables and constraint conditions can be used.

#### *Application of Random-Sampling Method*

Structural synthesis (ref. 61) can be defined as rational selection and improvement of a structural design configuration, in terms of weight or cost, without violating given failure conditions, manufacturing, or design limitations. The conventional way of designing an efficient structure is based on the designer's past experience, good

judgment, and trial and error until a satisfactory solution is found.

The basic rationale for applying random-sampling (RS) methods to structural synthesis problems is the similarity in method between an RS-type solution and the conventional solution, previously described. Another point in favor of an RS method is that it can be applied to almost any type of structural synthesis problem with little statistical theory. Finally, since the method is a completely random procedure, those using it cannot be handicapped by prejudices or oversights unless purposely biased by the programmer.

For the sake of illustration, let us assume that the structural part to be designed has three variables of thickness, spacing, and height, each with given limitations. This design can be expressed as  $X = X(\xi_1, \xi_2, \xi_3)$ , where  $\xi_1, \xi_2, \xi_3$  are thickness, spacing, and height, respectively. The requirements are given as the local and general stability for a given load; the merit function is the weight.

This problem can be solved by checking all possible design configurations, that is, all the distinct  $X$ 's for local and general instability, and selecting the one with the minimum weight among the stable configurations. However,  $\xi_1$  is a continuous variable that can take any value between  $\xi_{1\min}$  and  $\xi_{1\max}$  making the number of distinct  $X$ 's infinite. In practice, however, the interval  $(\xi_{1\min} - \xi_{1\max})$  can be divided into a finite number of slices, assuming that  $\xi_1$  is a variable that can take only a given number of values between minimum and maximum.

Assuming that thickness, spacing, and height can be divided into 100 slices each, the number of distinct design configurations is 1 million. Since there are two stability conditions in addition to weight, 3 million computations are the number of points theoretically to be checked. The function of the RS procedure is to cut the number of computations to an economical few hundred. Some of the sampling steps used are described below.

Importance sampling gives a way of biasing the random sampling so that some samples are drawn from zones where the probability of success is high, and less from zones where the probability of success is small. In other words, biasing is done to increase the probability that the sample will be drawn from an interesting region (ref. 62).

The combination of analytical and probabilistic solutions sometimes reduces the variance of the outcome; therefore, the optimum sample size is computed for some of the steps whenever that can be done without excessive effort.

If the density function of the random sample is approximated from the initial trial with respect to certain sections of the sample space, this function is used to assign a certain size of sample to each section for consecutive trials.

### Basic Screening Steps of the Program

The problem consists of locating a design point in the space of all possible design points, so that all the design requirements are satisfied and the evaluated merit function is as close to its optimum value as desired. Such a design point is denoted as  $X^*$ . Initially, then, we have

$$P(X=X^*)=P(X \in S^*) \quad \text{if } X \in S$$

$S$  is the  $n$ -dimensional space of design variables where  $n$  is the number of variables for the particular problem. The position of each design point  $X$  in this space is specified by the value of its coordinates  $\xi_k$ :

$$X=X(\xi_1, \xi_2, \dots, \xi_n)$$

The boundaries of the design space are specified by the minimum and maximum allowable values of the design variables. In the literature these boundaries are commonly referred to as side constraints.

In design problems the number of significant digits is limited for practical reasons; therefore, the random variable  $\xi_i$  can take only discrete values.

A design point is considered unacceptable if it violates any of the constraint conditions. Thus the only requirement for the  $g_i$  is that it must have a computable value for every  $X$  in  $S$ .

The concepts mentioned so far have been illustrated in a problem having two variables. (See fig. 60 from ref. 61.) The boundary between  $U$  and  $A$  is designated by  $G$ , which represents a hypersurface having concave and convex portions. By this program, random points are chosen and checked against the given constraints to determine whether they are in  $A$  or  $U$ ; this checking continues until no point in  $A$  can be found with a merit function lower than the previous one. The merit function  $F(X)$  is the function to be optimized. It has a unique value for every  $X$ , which is computable. To improve the probability of success with a minimum number of computations, a system of sampling techniques is utilized (described in ref. 56). This operation is based largely on the assumption that the evaluation of main constraints for a given  $X$  demands an effort much greater than the computation of merit function for that design point; therefore,  $X$  must be avoided as much as possible, and the information obtained from the merit function values must be fully used.

### COMPUTER-AIDED DESIGN OF STRUCTURES

Let us consider the possible utilization of computer capabilities to make design decisions more rapid and effective (ref. 53). In the past, engineers have carried out much of their design and practice

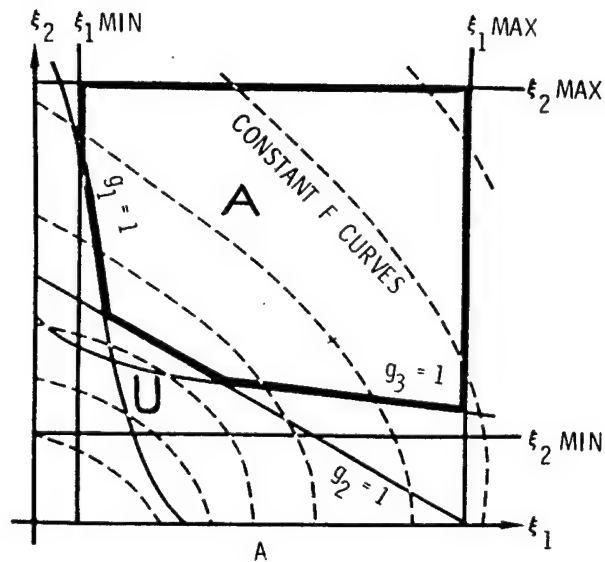
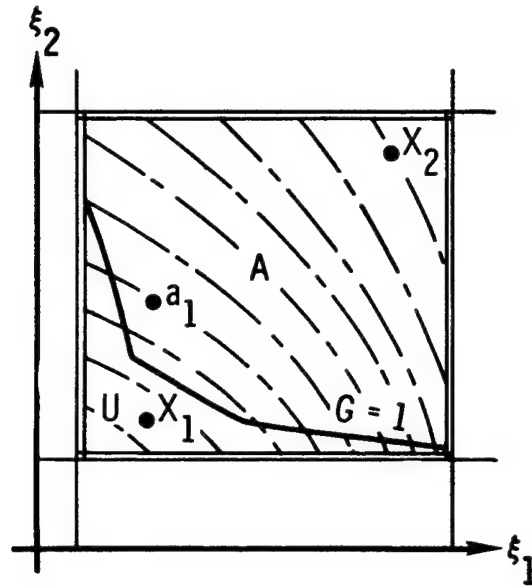


FIGURE 60.—Search for zone of optimum design. (Reprinted from *Proceedings 4th Aerospace Science Meeting*.) (Courtesy of American Institute of Aeronautics and Astronautics.)

efforts by analytical investigations using computers. The engineer, for example, determines the response of a given structure under applied loads and compares the behavior to allowable criteria. Generally, this design process involves a number of successive iterations. Although it is conceivable that a design can be made by direct solution of a close-form equation that expresses the primary design problem, the difficulties associated with the expression of design-problem parameters make use of this process very unlikely in the near future. Rather, the computer can be used as an effective design tool for analytical techniques, since it allows rapid synthesis by iteration.

In the past, designers often had the solutions to a limited number of discrete problems compiled from experience. With increased knowledge, experience, and the aid of high-speed, electronic digital computers, today's designer can execute the design process with greater effectiveness. Needless to say, many problems formerly beyond the designer's capabilities can now be solved.

In the preliminary design process, furthermore, several simplified techniques enable the computer to generate considerably more data

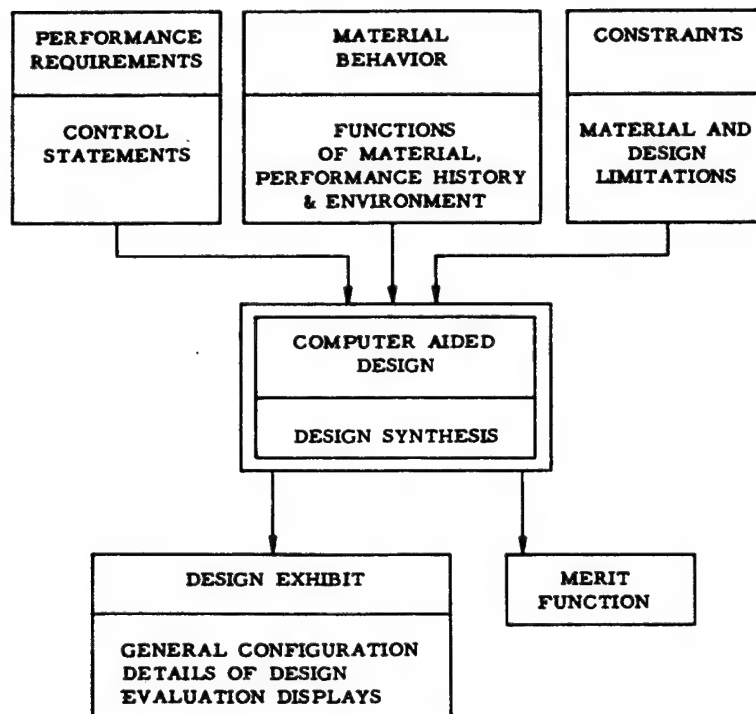


FIGURE 61.—General logic flow of computer-aided solution.



than the engineer. A computer can also exhibit these data in the form of drawings and specifications. Figure 61 shows the logic flow of computer-aided solutions. In table 2 (from ref. 53) a summary-comparison of computer-aided design procedures is given with applications to multistage launch vehicles, bridges, and domelike structures.

TABLE 2.—*Comparison of Computer-Aided Design Procedures*

Launch vehicle	Bridges	Domelike structures
<b>Performance requirements:</b> Velocity requirements. Specified payload weight in specific (orbital) mode. Type of mission (scientific vs. military; manned vs. unmanned).	Number of traffic lanes. Magnitude and distribution of loads. Clearance height for vehicles. Topographical conditions. Subsurface conditions. Arterial approach requirements.	Specified enclosed volume of surface area. Specified maintained environment in the enclosed volume. Specified esthetic requirements. Structural provisions to resist specified type, magnitude, and distribution of loads.
<b>Material behavior properties:</b> Strength/density relationships. Elastic/density relationships. Chemical compatibility factors.	Strength/density relationships. Elastic/density relationships. Corrosion-resistant properties.	Strength/density relationships. Elastic/density relationships. Absorptivity/emissivity properties. Thermal insulation/weight relationships. Acoustic insulation/weight relationships.
<b>Constraint functions:</b> Acceleration limits. Feasible stage diameters and fineness ratios. Time element for design (i.e., 1965 vs. 1970 type structure systems). Manufacturing-procurement feasibility of components.	Feasible geometric proportions. Navigation clearance. Minimum traffic lane width. Applicable specifications of the American Association of State Highway Officials. Gradient limitations. Dynamic response.	Minimum height as a function of distance from dome perimeter. Upper and lower bounds on temperature and humidity. Upper and lower bounds on acoustic characteristics. Fabrication limitations.

TABLE 2.—*Comparison of Computer-Aided Design Procedures—Con.*

Launch vehicle	Bridges	Domelike structures
Least cost components.	Aerodynamic response.	Maximum construction time.
Manufacturing, R&D time scheduling.	Fabrication limitations.	Construction time period.
Launch environment.	Construction period limitations (time and environment).	Amortization method.
Recovery problems.	Construction time period.	
Minimum gage restrictions.	Depreciation method.	
Scope of design investigation:		
Vehicle geometric models (configurations).	Variations in deck width as function of number of traffic lanes.	Variations in aspect ratios (height/radius at base).
Construction concepts (components).	Deck stacking concepts.	Variations in meridian curvature properties.
Variations in strength/density of materials.	Variations in span lengths.	Variations in framing geometry.
Effects of loadings induced by various trajectories.	Variations in support concepts.	Variations in framing material.
Pressure and temperature variations due to flight loadings, trajectories, and system changes.	Variations in profile gradients.	Variations in support concepts.
Performance weight partials.	Variations in anchorage configurations.	
Cost analysis (R&D plus Operational).	Variations in strength/density ratios of principal structural materials.	
Design exhibit:	Variations in ramp concepts.	
Profile drawings.	Cost effects upon adjacent land areas.	
Design sketches.		
Component detail sketches.	Drawings of bridge profile, cross sections, and elevation.	Drawings of dome cross section.
Master dimensions.	Detailed geometry of components.	Three-dimensional coordinate values of all space frame joints and other significant positions.
Parts lists.	Horizontal, vertical and torsional rigidities of bridge sections.	Internal loads and stresses in all members.
Cost analysis weight statements.	Dynamic response properties.	Load deflection of all significant joints.
Detailed weight statements.	Aerodynamic response properties.	Aerodynamic response properties.
Performance weight statements.	Parts list.	
Detailed geometry of components.	Excavation and	

TABLE 2.—*Comparison of Computer-Aided Design Procedures—Con.*

Launch vehicle	Bridges	Domelike structures
Design conditions (loads, pressures, temperatures). Evaluation logic in selection of candidate vehicle evaluation curves).	foundation costs. Material costs. Fabrication costs. Erection costs. Maintenance costs. Operational costs. Total construction time.	Excavation and foundation costs. Material costs. Fabrication costs. Erection costs. Maintenance costs. Operational costs. Total construction time.
Merit function: Cost per pound payload in specified earth orbit or space trajectory as a function of number of launches within particular time period.	Cost per year of operation per vehicle ton capacity.	Cost per year of operation per usable unit enclosed, operating in the specified environmental condition.

Finally, let us emphasize one aspect of the design process that has not yet received particular attention: proposed, or baseline requirement, changes. Often changes are made in design criteria, design philosophy, geometrical constraints, and/or environmental conditions for one or more reasons. These changes may result in revised engineering drawings and specifications and, perhaps, in additional tooling and testing. Whatever the effects of a given change are, however, the objective of that change must be met. Since detailed analyses cannot be made to check the results of a proposed change, a tool is needed to assess it rapidly and efficiently. This can be a digital computer programed to synthesize a structure for loads imposed on the body for a specific function and to calculate a specific parameter, or its changes, in terms of other suitable parameters.

## Appendix

### SELECTING MATERIALS FOR MINIMUM WEIGHT

Particularly in aerospace work (ref. 63), reduced weight means improved performance. Weight savings can also be important for autos, trucks, and railway cars, because a pound saved in the structure may permit a greater payload or increase general performance.

Since an aerospace vehicle cannot be designed for minimum weight alone, however, the designer must consider the environmental effects to which the structure may be subjected. No single material or construction can retain superior strength over the entire range of potential loads and temperatures. The optimum structure must consist of many materials, each suited for a particular job. In addition to strength and stiffness, the materials must be evaluated for cost, ease of fabrication, toughness, durability, and other properties.

#### *Weight Index for Stiffness*

Geometry is particularly critical in structures designed for stability and stiffness. Although stiffness will change with the geometry of the structure, the efficiency of a given geometry is proportional to the merit-weight index. This index can be calculated using the ratio of a material's modulus of elasticity to density ( $E/\rho$ ).

In an aerospace vehicle, increases in stiffness are accompanied by increases in structural stability and natural frequencies. Thus, a high merit-weight index for stiffness ( $E/\rho$ ) would indicate that the structure can efficiently handle static and dynamic problems of elasticity (e.g., aerodynamic response and flutter), acoustics (e.g., vibration fatigue), as well as load-carrying capacity.

#### *Weight Index for Tensile Loads*

Selection of the optimum material and construction is often made easier by using merit-weight indices. In a structure subject to short-time tensile loading, for example, the weight is inversely proportional to the  $(\sigma_{\text{allow}}/\rho)$  index, which is the ratio of allowable tensile stress to the density of the structure. This index is commonly known as the merit-weight index for tensile criteria.

An evaluation of the merit-weight index for tensile conditions as a function of temperature and time is shown in figure 62 (from ref. 63) for a few typical materials. This illustrative guide shows that an increase in time at temperature is equivalent to shifting the abscissa (time-temperature parameter) to the right, thus resulting in a decrease in strength.

The merit-weight index of a particular material is determined by the intersection of the curve with a vertical line, the lower end of which passes through the intersection of the appropriate time and temperature lines. In figure 62, which gives a typical example of how this index is obtained, titanium is shown to be the lightest of the materials considered for withstanding a tensile load after exposure to 800°F for 100 hrs.

Plots similar to those shown in figure 62 can also be made for other merit indices, such as allowable rupture stress/density ( $\sigma_r/\rho$ ), or allowable creep stress/density ( $\sigma_c/\rho$ ) in areas of constant stress, such as the powerplant.

Except for orthotropic constructions such as filament-wound pressure vessels, the geometry of tensile-loaded structures is not too critical. This is generally true provided that good design practices, such as avoiding stress concentrations, are observed.

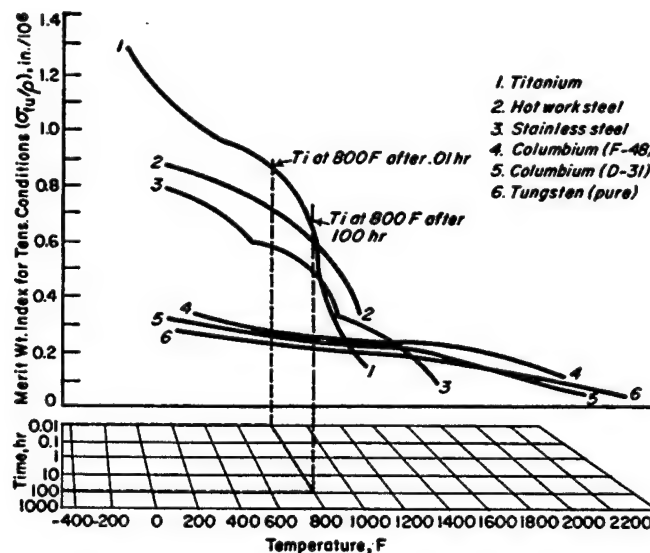


FIGURE 62.—Merit-weight indices of materials under tensile loading as a function of temperature and time. (Courtesy of Materials in Design Engineering.)

### Weight Index for Compressive Loads

The weight efficiency of materials carrying compressive loads can be expressed by a merit-weight index for stability ( $\sigma_{cr}/\rho$ ). Here  $\sigma_{cr}$  is the instability stress depending on the geometry of the structure, the load, and stress-strain relationship of the material.

Evaluations can be made independent of structural size by using weight ( $W/b^2$ ) and load ( $P/b^2$ ) indices, where  $W$  represents the weight per inch of the structural cross section,  $P$  represents the load, and  $b$  the characteristic dimension of the structure (e.g., width of plate or length of column). Thus, as shown in figures 63 and 64 (from ref. 63), typical plots of these indices can be derived for various materials and structural configurations. Such plots are an effective tool for selecting the right material and/or structural configuration.

#### Unstiffened Plate

A comparison of the weight and load indices (in the form of a log-log plot) of typical engineering materials is shown in figure 63. By mathematical analysis, the weight index of a material is proportional to the cube root of the load index at low load indices. The relationships can be represented by straight lines with a slope of 1/3 as a result of comparing materials. Thus, at low load indices, the merit index or efficiency

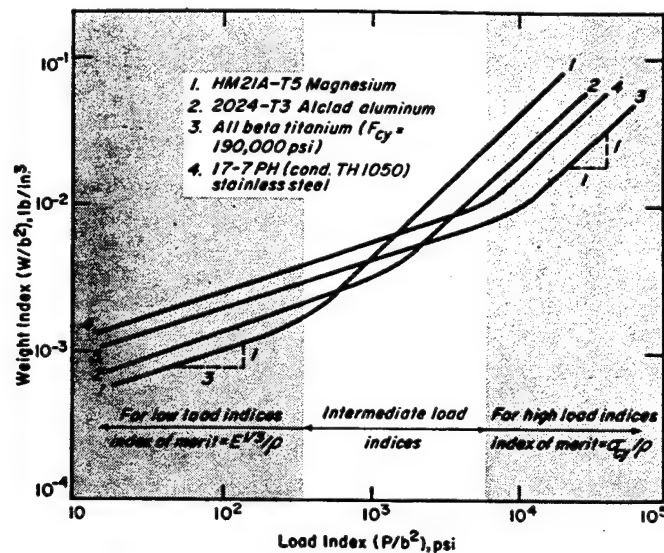


FIGURE 63.—Weight-load comparison of flat unstiffened compression panels at room temperature. (Courtesy of Materials in Design Engineering.)

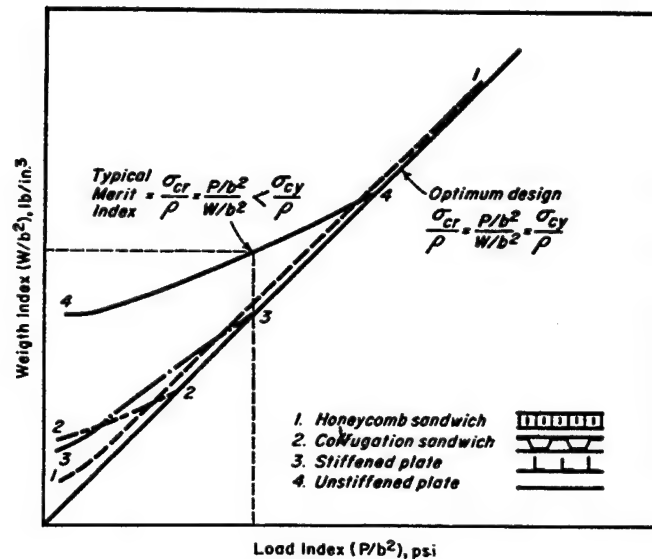


FIGURE 64.—Weight efficiencies of four configurations. Comparison is generalized for any material. (Courtesy of Materials in Design Engineering.)

of a material can be calculated by  $(E^{1/3}/\rho)$ . (For double-faced corrugations, instead of unstiffened plate, the merit index or efficiency of a material would be  $(E^{1/3}/\rho)$ .)

For high load indices, the straight lines assume a slope of 1/1, and the weight index is directly proportional to the load index. In this area the efficiency of the material can be measured by the allowable compressive yield stress/density  $(\sigma_{cy}/\rho)$ . This index can be used for all types of construction.

#### Types of Construction

A guide to the weight efficiency of four typical structural configurations is given in figure 64. This comparison can be generalized for any material used in stiffened or unstiffened flat plate, honeycomb, or corrugated sandwich form.

For a given load in figure 64 the construction having the best weight efficiency is indicated by the lowest vertical ordinate. For any of the four constructions, the efficiency can be calculated by the merit index  $(\sigma_{cr}/\rho)$  (allowable stability stress/density), which is equal to the abscissa divided by the ordinate,  $P/b^2:W/b^2$ . This figure shows that the optimum stress efficiency is  $(\sigma_{cy}/\rho)$  (allowable compressive yield stress/density). Each of the four constructions approaches the optimum value asymptotically with increasing load indices.

Figure 64 also shows that at low load indices, a honeycomb sandwich

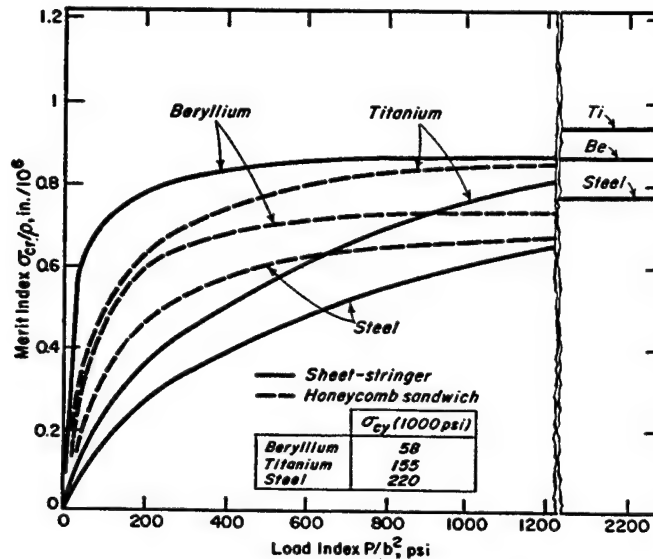


FIGURE 65.—Efficiency of beryllium, steel and titanium in two types of constructions at room temperature. (Courtesy of Materials in Design Engineering.)

construction is most efficient, since it is capable of stabilizing the faces to yield, although it suffers a slight weight penalty due to core and bonding materials. Keep in mind, however, that variations in efficiency can occur at very low loads because of minimum gage and fabrication requirements.

Next in weight efficiency is the corrugated sandwich, in which the stabilizing stress rapidly approaches the asymptotic yield stress. The stiffened steel-plate construction, in which efficiency increases with the number of stiffeners used, is next in efficiency; whereas the unstiffened plate is the least efficient structure, since it requires the largest load index to attain the yield stress. The relative efficiency of honeycomb sandwich can be offset by an extremely high stiffness merit index ( $E/\rho$ ), such as is exhibited by beryllium.

Additional weight indices, given in figure 65 (from ref. 63), show the efficiency of steel, titanium, and beryllium when used in a honeycomb-sandwich or sheet-stringer construction. This study is far from complete; additional materials, configurations, and temperatures should be considered before making a final selection.

#### Other Important Factors

In choosing material and configuration for minimum weight, allowance must also be made for factors such as tolerances, available gages, and required design details (e.g., joints and reinforcements). In



addition, the physical properties of the material have to be considered, and compromises often must be made in determining the best combination of properties.

For example, a low thermal expansion coefficient,  $\alpha$ , is desirable for reducing thermal distortion. Similarly, a low product of elastic modulus and thermal expansion coefficient,  $E\alpha$ , will help reduce thermal stresses. Reductions in thermal gradients that result in thermal stresses require materials with a high thermal conductivity ( $k$ ), specific heat ( $c$ ), emissivity ( $e$ ), or diffusivity ( $k/c_p$ ).

Although minimum weight analyses are relatively simple when a monolithic structure is used, these procedures become more complex with composite structures. An aerospace vehicle, for example, may consist of a hot structure that supports the applied loads near the equilibrium temperature, or a protected structure which consists of a thermal protection system to resist local air loads, while maintaining an efficiently lower temperature on the load-carrying structure. When designing such a composite structure, consequently, tradeoff studies must be made to determine what combination of thermal protection and load-carrying structure will provide the minimum weight.

### CALCULATING WEIGHT EFFICIENCIES OF PRESSURE VESSELS

Although unfired pressure vessels are often selected without regard to weight, many applications require them to be as light as possible. When low vessel weight is needed, it is important to know how the relative weight efficiencies of different materials compare when they are used in such common shapes as cylinders, spheres, and oblate spheroids.

#### *Material and Shape Comparison*

A basic formula that can be used in evaluating the weight efficiency of pressure vessel materials is

$$\frac{PV}{W} = \frac{1}{K} \left( \frac{\sigma}{\rho} \right)$$

In essence, this formula tells us that the product of pressure  $P$  and volume  $V$  divided by weight  $W$  is theoretically equal to a shape factor  $1/K$  times the strength-to-density ratio ( $\sigma/\rho$ ) of the pressure vessel material.

So as to make a direct weight comparison for vessels of the same pressure-volume ( $PV$ ) capability, this formula can be rearranged as follows:

$$W = K \left( \frac{\rho}{\sigma} \right) PV$$

Since we want to keep the  $PV$  product constant when comparing materials, the formula simplifies to:

$$W \propto K \left( \frac{\rho}{\sigma} \right)$$

This formula tells us that for a given pressure-volume value, the vessel weight for a material is equal to its density/strength times a shape factor. The shape factor ( $K$ ) is 2 for cylinders, 1.5 for spheres, and 3 for oblate spheroids.

This last formula can be used to make a side-by-side comparison, such as that shown in figure 66 (from ref. 63), which lists the relative weights, at constant pressure and volume, of three basic shapes fabricated from fiber glass, titanium, aluminum, and high-strength steel. In this chart, the relative weight values are calculated simply by taking the weight of the lightest vessel (S fiber glass) and dividing all other weight values by it. Thus, the following formula can be used to find the relative weight for a cylinder of aluminum (A) compared with S fiber glass (F):

$$\frac{W_A}{W_F} = \frac{2(\rho_A/\sigma_A)PV}{2(\rho_F/\sigma_F)PV} = \left( \frac{\rho_A}{\sigma_A} \right) \left( \frac{\sigma_F}{\rho_F} \right) = 2.9$$

Naturally, in a relative weight evaluation, several assumptions have to be made in establishing the appropriate strength values.

#### Establishing Strength Values

The strength values for fiber glass in figure 66 are based on total wall composite strength (adjusted to vessel axis) computed from room-temperature burst tests on 4-in. balanced cylinders, 8-in. spheroids, and 17-in. spheres. Strength values will be different for other vessels, depending on their sizes and proportions; however, scaling factors can be calculated.

The strength of virgin glass filaments is much higher than the strengths actually achieved when these filaments are used in pressure vessels. Nevertheless, all other considerations aside, fiber-glass vessels have a higher strength-to-density ratio than metals. Ultimate strength, rather than yield strength, is used here because some fabricators have the capability to provide the higher strength levels. However, care must be exercised in selecting the spread between yield and ultimate strengths of the pressure-vessel materials.

In the calculations for relative weight, allowance has not been made for the beneficial effect of biaxial reinforcement under load for homogeneous metals. Allowance for the biaxial reinforcement effect may

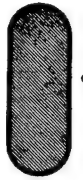


Material		S Fiber glass (20 end-HTS finish epoxy-anhydride resin)	E Fiber glass (20 end-HTS finish epoxy-anhydride resin)	Titanium (6 Al-4V, forged & heat treated)	Aluminum (7075-T6)	High Strength Steel (Ladish D6A-C, heat treated)
Vessel Shape	Cylinder  $W = 2 \left( \frac{\rho}{\sigma_{hoop}} \right) PV$	1.0 $\frac{\sigma_{hoop}}{\rho} = \frac{170,000}{0.073} = 2.33 \times 10^6$	1.4 $\frac{\sigma_{hoop}}{\rho} = \frac{125,000}{0.075} = 1.67 \times 10^6$	2.3 $\frac{\sigma}{\rho} = \frac{165,000}{0.16} = 1.03 \times 10^6$	2.9 $\frac{\sigma}{\rho} = \frac{76,000}{0.097} = 0.8 \times 10^6$	3.0 $\frac{\sigma}{\rho} = \frac{220,000}{0.283} = 0.78 \times 10^6$
	Oblate Spheroid  $W = 3 \left( \frac{\rho}{\sigma_{long}} \right) PV$	1.0 $\frac{\sigma_{long}}{\rho} = \frac{245,000}{0.073} = 3.36 \times 10^6$	1.4 $\frac{\sigma_{long}}{\rho} = \frac{185,000}{0.075} = 2.47 \times 10^6$	—	—	—
	Sphere  $W = 1.5 \left( \frac{\rho}{\sigma} \right) PV$	1.5 $\frac{\sigma}{\rho} = \frac{85,000}{0.073} = 1.16 \times 10^6$	2.1 $\frac{\sigma}{\rho} = \frac{65,000}{0.075} = 0.87 \times 10^6$	1.7 $\frac{\sigma}{\rho} = \frac{165,000}{0.16} = 1.03 \times 10^6$	2.2 $\frac{\sigma}{\rho} = \frac{76,000}{0.097} = 0.8 \times 10^6$	2.2 $\frac{\sigma}{\rho} = \frac{220,000}{0.283} = 0.78 \times 10^6$

FIGURE 66.—Relative weights for unlined pressure vessels. (Courtesy of Materials in Design Engineering.)

provide as much as a 15-percent reduction in the relative weights that are indicated for homogeneous metal vessels.

The differences in weight of thermal insulations required for glass or metal vessels are not especially significant for some applications, such as rocket motor cases. Unlike a metal vessel, however, a glass vessel for gas storage requires a sealant liner that may vary in weight from 5 percent (for thick-walled vessels) to as much as 30 percent (for extremely thin-walled vessels) of the structural membrane weight.

Correction factors should be made in allowable design stresses of the various materials to compensate for the effect of extended internal pressure cycling or holding time. The degree to which these factors become significant depends on the specific application.

Allowance should also be made for all structural loads other than those imposed by internal pressure. In such cases, factors such as compressive strength, modulus of elasticity, and other material constants (as applied to buckling relationships) become important. These additional factors can change the comparisons which have been made here for conditions of simple internal pressure loading.

## EXAMPLES OF COMPUTER-AIDED DESIGN OF STRUCTURES

### *Space-Frame Dome*

The design requirements for a space-frame dome (ref. 53) include: a certain volume, specified (or range) height-to-radius ratio(s), load conditions, and environmental conditions. Common variations in dome surfaces are shown in figure 67 (from ref. 53).

By using computer-aided design techniques, a study of the effects of perimeter discontinuities along with wind-load distributions, irregular loads, thermal stresses, and support conditions can be performed simultaneously. These computer techniques have been developed to analyze space frames by using the principle of minimum-strain energy. In addition to the geometry and wind-load inputs, structural parameters, such as cross-sectional areas of members, moments of inertia, and connection rigidities, are coded into the computer program so that they may be easily revised for design optimization. These programs permit summing of the total inter-strain energy of the structures, case by case, together with all the interaction products, so that required influence coefficients and structural deflections may be computed.

The design display may include such items as: a graphic profile of each configuration investigated, three-dimensional coordinate values of all space-frame joints and other significant positions, internal loads and stresses in all members, deflection of all significant joints, cost data, and construction time, to mention a few.

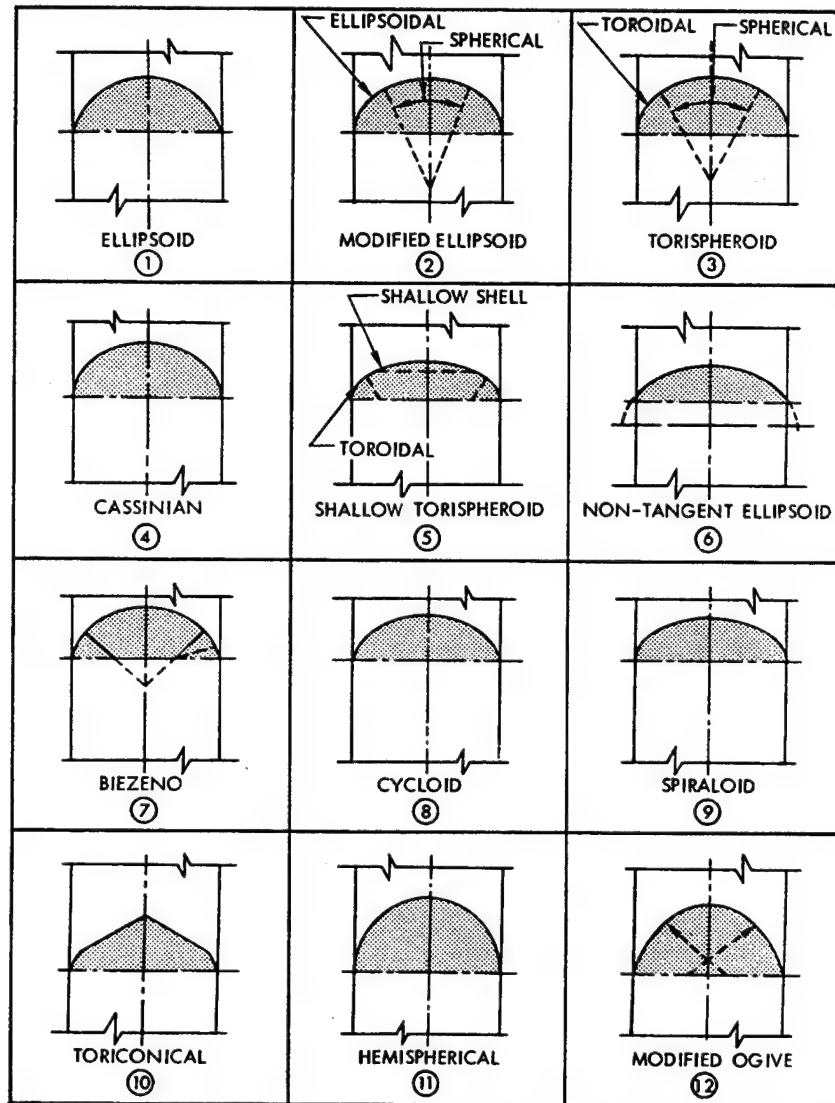


FIGURE 67.—General categories of surfaces of revolution analyzed by computer subroutines.

A typical geometrical configuration resulting from a computer-aided design study is shown in figure 68 (from ref. 53). In this study, the base radius and dome height were input data; load magnitudes and distributions were also specified. The merit function used in this study was defined as "cost-in-place," expressed in terms of enclosed volume. The configuration, called "Geolatic Framing," resulted

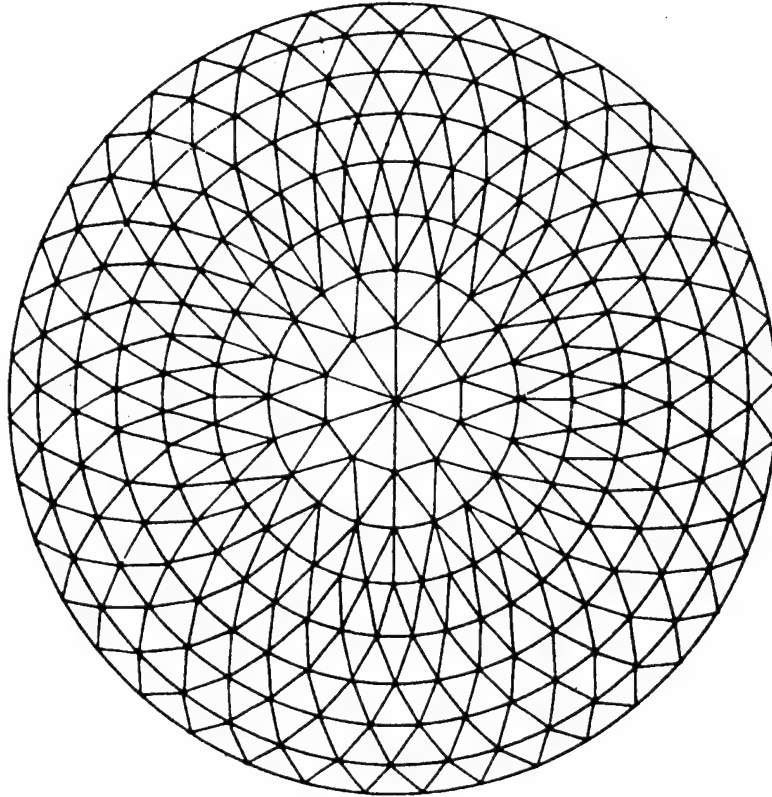


FIGURE 68.—Dome-framing plan developed by computer-aided design techniques.

from a study at North American Aviation, Inc., relating to the application of computer techniques to civil structures. "Geolatic" refers to framing systems in which a considerable number of framing members are in parallels of latitude. In the configuration shown, approximately 60 percent of the framing members were identical in fabrication.

#### ***Bridge Design***

Another example of a computer-aided structural design is a bridge which spans a given distance and provides a specified traffic-flow capability. The performance requirements may include the number and width of traffic lanes; magnitude and distribution of imposed loads (static, dynamic, aerodynamic, seismic); and arterial-approach geometry. Constraint functions may include minimum clearance below the bridge, maximum allowable profile products, and applicable

Government specifications. Outlines of general bridge concepts are shown in figure 69 (from ref. 53).

The investigation of each proposed configuration should include principal variations in truss-framing geometry, pier locations, deck-framing concepts, approach ramp effects, material usage, maintenance

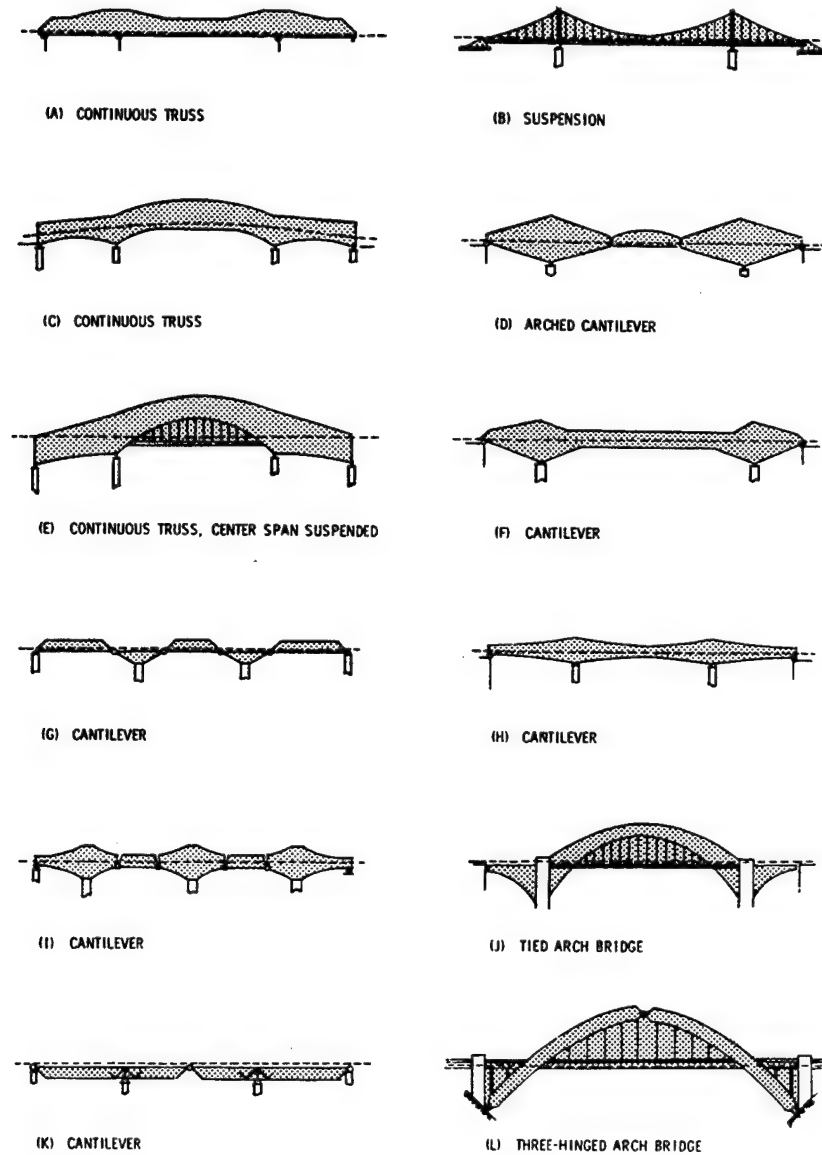


FIGURE 69.—Representative bridge configuration available for evaluation.

concepts, and related problems. An overall merit function can be expressed for each proposed design to indicate the total cost of ownership and operation per year in terms of vehicle-ton capacity. Output data may include: graphic profile of each configuration investigated, including cross sections and elevations; detailed geometry of components; rigidities of bridge sections; aerodynamic and dynamic response properties; cost data; and construction time, among others.

### EXAMPLE OF STRUCTURAL OPTIMIZATION

The purpose of this analysis (ref. 64) is to establish a procedure for optimizing an integral stringer and ring-stiffened shell subjected to axial load. (See fig. 70 from ref. 64.) Two modes of failure are considered: strength based on the Von Mises yield criteria and elastic instability. The elastic instability consists of general instability (overall collapse of the cylinder), buckling of the unsupported panel lengths between rings, buckling of the skin bounded by the ring and stringers, and crippling of the outstanding stringer rib.

The optimization procedure is based upon elastic buckling with the following parameters being optimized: depth of rib, skin thickness, rib thickness, rib spacing, and ring spacing. The following assumptions are made:

- (1) Internal pressure has no effect on the overall general instability.
- (2) Ring spacing is sufficiently close so that the rings and skin are equally stressed.
- (3) Curved panels are treated as flat plates, since the ribs are closely spaced.
- (4) Critical buckling stresses are within the elastic limit.

To minimize the number of design parameters, two relationships are established:

- (1) The depth of the ring is  $2\frac{1}{2}$  times that of the longitudinal stringer. This depth is arrived at by equating the local crippling stress of the outstanding leg of the longitudinal stringer with that of the ring web:

$$k_s \frac{E}{1-\mu^2} \left( \frac{t_w}{b_s} \right)^2 = k_r \frac{E}{1-\mu^2} \left( \frac{t_w}{K_1 b_w} \right)^2$$

where

$k_s = 0.385$  (one edge free)

$k_r = 3.29$  (both edges simply supported)

and  $K_1$  equals 2.92; however, since one of the edge conditions of the web is actually elastically supported, use  $K_1 = 2.5$ . Therefore, depth of ring equals  $2.5b_w$ .

- (2) By equating the local crippling stresses of the outstanding leg



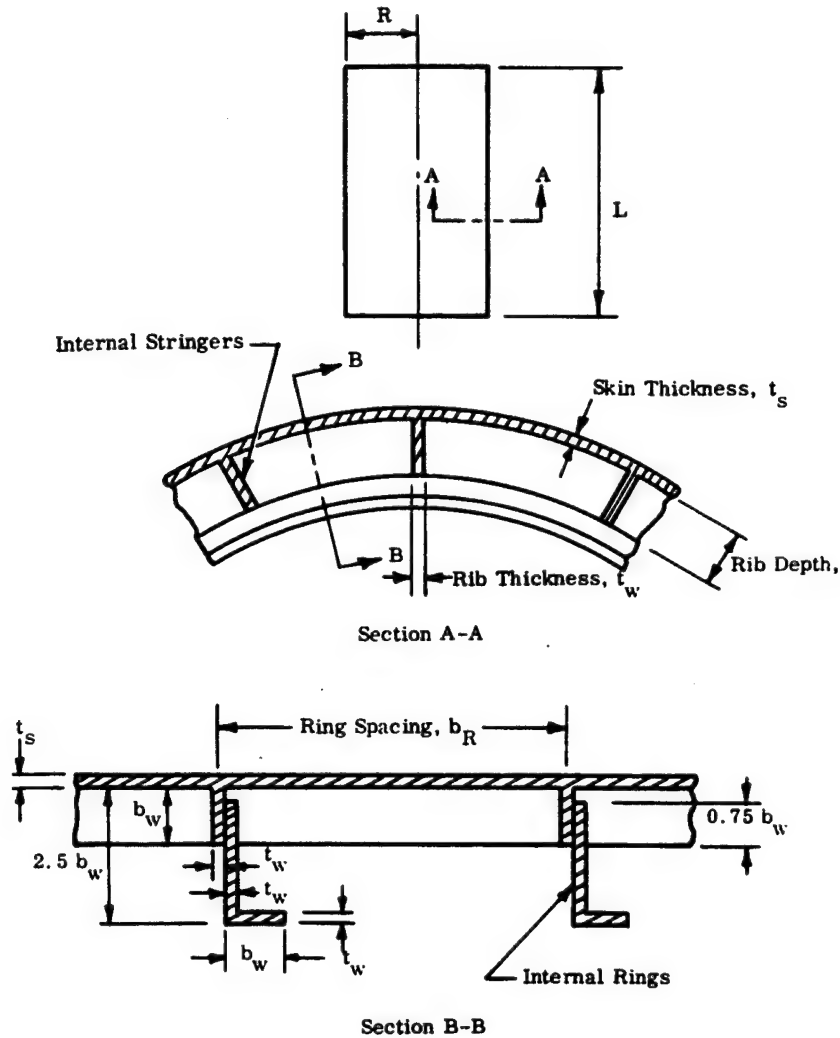


FIGURE 70.—Integral stringer and ring-stiffened cylinder geometry.

of the longitudinal stringer with that of the ring flange, we obtain a flange width equal to that of the stringer depth.

#### **Failure Modes**

##### **Buckling Criteria**

In predicting general instability, the equations developed by Block, Card, and Mikulas (ref. 65) are used. These equations represent the latest state of the art in buckling of orthotropic cylinders and take into consideration the effects of asymmetry; i.e., the effect of whether

the rings and stringers are located on the inside or outside of the skin. The equations are:

$$N_z \frac{L^2}{\pi^2 D} = m^2 (1 + \beta^2)^2 + m^2 \frac{EI_s}{dD} + m^2 \beta^4 \frac{EI_r}{ID} + \left( \frac{G_s J_s}{dD} + \frac{G_r J_r}{ID} \right) m^2 \beta^2 + \frac{12Z^2}{m^2 \pi^4} \left( \frac{1 + \bar{S} \Lambda_s + \bar{R} \Lambda_r + \bar{S} \bar{R} \Lambda_{rs}}{\Lambda} \right)$$

where

$$\Lambda_r = 1 + 2\alpha^2 \beta^2 (1 - \beta^2 \mu) \frac{\bar{Z}_r}{R} + \alpha^4 \beta^4 (1 + \beta^2)^2 \left( \frac{\bar{Z}_r}{R} \right)^2$$

$$\Lambda_s = 1 + 2\alpha^2 (\beta^2 - \mu) \frac{\bar{Z}_s}{R} + \alpha^4 (1 + \beta^2)^2 \left( \frac{\bar{Z}_s}{R} \right)^2$$

$$\Lambda_{rs} = 1 - \mu^2 + 2\alpha^2 \beta^2 (1 - \mu^2) \left( \frac{\bar{Z}_r}{R} + \frac{\bar{Z}_s}{R} \right) + \alpha^4 \beta^4 [1 - \mu^2 + 2\beta^2 (1 + \mu)] \left( \frac{\bar{Z}_r}{R} \right)^2 + 2\alpha^4 \beta^4 (1 + \mu)^2 \frac{\bar{Z}_r \bar{Z}_s}{R^2} + \alpha^4 \beta^2 [2(1 + \mu) + \beta^2 (1 - \mu^2)] \left( \frac{\bar{Z}_s}{R} \right)^2$$

$$\Lambda = (1 + \beta^2)^2 + 2\beta^2 (1 + \mu) (\bar{R} + \bar{S}) + (1 - \mu^2) \cdot [\bar{S} + 2\beta^2 \bar{R} \bar{S} (1 + \mu) + \beta^4 \bar{R}]$$

with

$$Z^2 = \frac{L^4 (1 - \mu^2)}{R^2 t^2}$$

$$\bar{S} = \frac{A_s}{td}$$

$$\alpha = \frac{m\pi R}{L}$$

$$D = \frac{Et^3}{12(1 - \mu^2)}$$

$$\bar{R} = \frac{A_r}{tl}$$

$$\beta = \frac{nL}{m\pi R}$$

In order to use the previously defined equations, the basic stability equation must be minimized with respect to  $m$  and  $n$  to obtain the minimum allowable loading. Because of the complexity and time limitation involved, however, it is assumed that the ring and stringer eccentricities do not affect the buckling-mode shape. Based on this

assumption, which has been subsequently proved to be valid, the equations for determining the buckling-mode shape for the Becker equation (ref. 64) are used. By utilizing the Becker equation to determine the buckling-mode shape and to nondimensionalize the design parameters, the following equations are obtained, letting

$$t_s = C_1 b_w$$

$$t_w = C_2 b_w$$

$$b_s = C_3 b_w$$

$$b_r = C_4 b_w$$

$$N_x = f(C_1, C_2, C_3, C_4) \frac{2ECb_w^2}{R}$$

where

$$f(C_1, C_2, C_3, C_4) = \frac{1}{2\phi^{1/2}} \left[ \frac{(1+\beta^2)^2 C_1^3}{12(1-\mu^2)} + \frac{C_2}{12C_3} + \beta^4 \frac{2.92C_2}{C_4} \right. \\ \left. + 0.375\beta^2 \left( \frac{C_2^3}{3C_3} + \frac{1.166C_2^3}{3C_4} \right) \right] \\ + \frac{\phi^{1/2}}{2} \left( \frac{1 + \bar{S}\Lambda_s + \bar{R}\Lambda_r + \bar{S}\bar{R}\Lambda_{rs}}{\Lambda} \right)$$

$$\phi = (d_{11} + 0.375\beta^2 d_{33} + \beta^4 d_{22}) \left( \frac{1}{a_{22}} + \frac{0.375\beta^2}{a_{33}} + \frac{\beta^4}{a_{11}} \right)$$

$$\beta^2 = P + (P^2 + Q)^{1/2}$$

$$P = \frac{a_{33}}{a_{22}} \left( \frac{a_{22}d_{11} - a_{11}d_{22}}{a_{11}d_{22} - 2a_{33}d_{33}} \right)$$

$$Q = \frac{a_{11}}{a_{22}} \left( \frac{a_{22}d_{11} - 2a_{33}d_{33}}{a_{11}d_{22} - 2a_{33}d_{33}} \right)$$

If  $\beta^2$  is negative, set  $\beta^2 = 0$

If  $(P^2 + Q)$  are negative, set  $\beta^2 = 0$

$$\bar{S} = \frac{C_2}{C_3 C_1}$$

$$\bar{R} = \frac{4.25C_2}{C_4 C_1}$$

$$\Lambda_s = 1 + \frac{2(\beta^2 - \mu)\bar{\zeta}_s \bar{\psi}_s}{\phi^{1/2}} + \frac{(1 + \beta^2)^2 \bar{\zeta}_s^2 \bar{\psi}_s^2}{\phi}$$

$$\Lambda_r = 1 + \frac{2\beta^2(1 - \beta^2\mu)\bar{\psi}_r \bar{\zeta}_r}{\phi^{1/2}} + \frac{\beta^4(1 + \beta^2)^2 \bar{\zeta}_r^2}{\phi}$$

$$\Lambda_{rs} = 1 - \mu^2 + \frac{2\beta^2(1-\mu^2)}{\phi^{1/2}} (\psi_r \bar{\xi}_r + \psi_s \bar{\xi}_s) + \frac{\beta^4[1-\mu^2+2\beta^2(1+\mu)]\bar{\xi}_r^2 \psi_r^2}{\phi} \\ + \frac{2\beta^4(1+\mu)^2 \psi_r \bar{\xi}_r \psi_s \bar{\xi}_s}{\phi} + \frac{\beta^2[2(1+\mu)+\beta^2(1-\mu^2)]\bar{\xi}_s^2 \psi_s^2}{\phi}$$

$$\Lambda = (1+\beta^2)^2 + 2\beta^2(1+\mu)(\bar{R} + \bar{S}) + (1-\mu^2)[\bar{S} + 2\beta^2\bar{R}\bar{S}(1+\mu) + \beta^4\bar{R}]$$

$$d_{11} = \frac{1}{C_3} \left[ \frac{C_2}{3} + \frac{C_1 C_2}{2} + \frac{C_1^2 C_2}{4} - \frac{C_2^2(1+C_1)^2}{4(C_2+C_3 C_1)} \right]$$

$$d_{22} = \frac{i_r + a_r (\bar{\xi}_r - \bar{Y})^2 + C_1 C_4 (\bar{Y})^2}{C_4}$$

$$a_r = 4.25 C_2$$

$$i_r = 2.92 C_2$$

$$\bar{\xi}_r = 1.44$$

$$\bar{\xi}_s = \frac{1+C_1}{2}$$

$$\bar{Y} = \frac{6.11 C_2}{4.25 C_2 + C_1 C_4}$$

$$d_{33} = \frac{C_2^3}{16 C_3} + \frac{0.219 C_2^3}{C_4} + \frac{C_1^3}{16}$$

$$a_{11} = C_1 + \frac{C_2}{C_3}$$

$$a_{22} = \frac{4.25 C_2 + C_1 C_4}{C_4}$$

$$a_{33} = \frac{3 C_1}{8}$$

$$D_{11} = d_{11} E b_w^3$$

$$D_{22} = d_{22} E b_w^3$$

$$D_{33} = d_{33} E b_w^3$$

$$A_{11} = a_{11} E b_w$$

$$A_{22} = a_{22} E b_w$$

$$A_{33} = a_{33} E b_w$$

$C$ , the buckling correction factor, equals 0.58.

Because no test data are available for this type of construction, the same buckling correction factor will be used as for the single-face corrugation (ref. 64).

In predicting the buckling of the unsupported panel lengths between rings, the same equation used for general instability will be used, with, of course, the stiffness of the circumferential rings being taken as zero.

The equation is

$$\frac{N_x^p l^2}{\pi^2 D} = m^2(1+\beta^2)^2 + m^2 \frac{EI_s}{dD} + m^2 \beta^2 \frac{GJ_s}{dD} + \frac{12Z^2}{m^2 \pi^4} \left[ \frac{1 + \bar{S} \Lambda_s}{(1+\beta^2)^2 + 2\bar{S}\beta^2(1+\mu) + \bar{S}(1-\mu^2)} \right]$$

In predicting the theoretical panel buckling load, the above equation must be minimized with respect to  $m$  and  $n$ . In order to simplify the minimization, a value of 1 is used for  $m$ , the number of buckling half wavelengths between rings. This is analogous to the buckling-wave pattern of a simply supported Euler column between rings. In order to minimize with respect to  $n$ , a numerical iteration scheme is used to obtain the minimum value of  $N_x^p$ . (See fig. 71 from ref. 64.)

To do this, let

$$\begin{aligned} m &= 1 \\ l &= C_4 b_w \\ d &= C_3 b_w \end{aligned}$$

so that

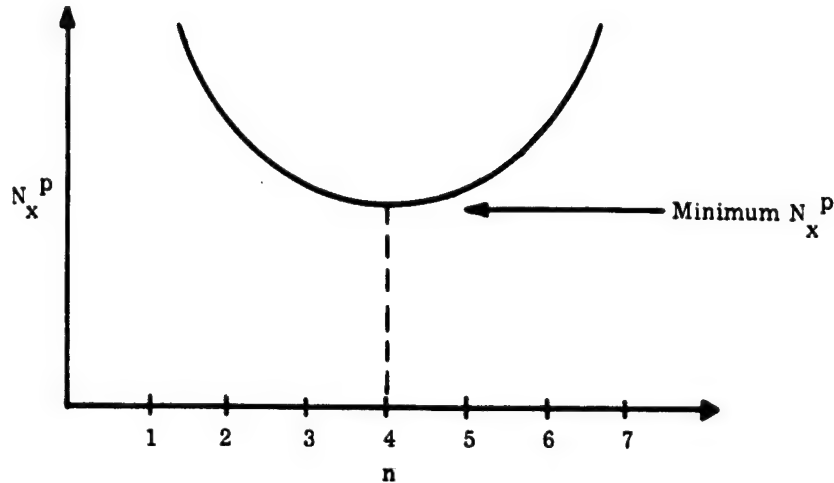


FIGURE 71.—Minimum value of  $N_x^p$ .

$$N_x^p = \left\{ \frac{\pi^2 D (1 + \beta^2)^2}{C_4^2 b_w^2} + \frac{\pi^2 C_2 E b_w}{12 C_4^2} + \beta_2 \frac{\pi^2 C_2^3 E b_w}{8 C_4^2 C_3} \right. \\ \left. + \frac{12 D Z^2}{C_4^2 \pi^2 b_w^2} \left[ \frac{1 + \bar{S} \Lambda_s}{(1 + \beta^2)^2 + 2 \bar{S} \beta^2 (1 + \mu) + \bar{S} (1 - \mu^2)} \right] \right\} C_p$$

$$\beta = \frac{n C_4 b_w}{\pi R}$$

$$\alpha = \frac{\pi R}{C_4 b_w}$$

$$\bar{S} = \frac{C_2}{C_1 C_3}$$

$$\bar{R} = \frac{4.25 C_2}{C_4 C_1}$$

$$D = \frac{C_1^3 E b_w^3}{12 (1 - \mu^2)}$$

$$Z^2 = \frac{C_4^4 (1 - \mu^2) b_w^2}{C_1^2 R^2}$$

$$\Lambda_s = 1 + \frac{\pi^2 R}{C_4^2 b_w} (\beta^2 - \mu) (1 + C_1) \psi_s + \frac{\pi^4 R^2}{C_4^4 b_w^2} (1 + \beta^2)^2 \frac{(1 + C_1)^2}{4} \psi_s^2$$

$C_p$ , the buckling correction factor, equals 0.58, which is the same factor used for general instability.

Assuming simply supported edge conditions and an aspect ratio of infinity, the critical rib-crippling stress (ref. 66) is

$$\sigma_{cr} = 0.385 \frac{E}{1 - \mu^2} C_2^2$$

Assuming simply supported edge conditions and an aspect ratio of infinity, the critical skin buckling (ref. 66) is

$$\sigma_{cr} = 3.29 \frac{E}{1 - \mu^2} \left( \frac{C_1}{C_3} \right)^2$$

#### Strength Criteria

In order to determine the maximum stress level in the skin, a modified form of the Von Mises yield equation is used. The skin is investigated, since its resultant stress will always be greater than, or equal to, that of the stiffening elements

$$\sigma = \sqrt{\left( \frac{N_x}{a_{11} b_w} \right)^2 - \left( \frac{N_x N_y}{a_{11} a_{22} b_w^2} \right) + \left( \frac{N_y}{a_{22} b_w} \right)^2}$$

**Optimization Procedure**

Optimum design parameters  $C_1$ ,  $C_2$ ,  $C_3$ ,  $C_4$ , and  $b_w$  must be determined to obtain a minimum-weight configuration. The approach to be taken is the concept of maximum strength-to-weight ratio. A logical range of  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  will be investigated and the corresponding strength-to-weight ratios calculated. The configuration with the maximum ratio will be investigated for panel buckling and for local forms of instability (skin buckling and rib crippling). If any of these forms of instability are violated, the values of  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  with the next highest strength-to-weight ratio are investigated. This process is continued until all forms of instability have been satisfied. Having determined the optimum values of  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$ , the value of the rib depth can be calculated to satisfy general instability using:

$$b_w = \sqrt{\frac{N_z R}{2CE[f(C_1, C_2, C_3, C_4)]}}$$

In determining the strength-to-weight ratios, the following equations are required:

$$\text{Average thickness, } t_{ave} = g(C_1, C_2, C_3, C_4)b_w$$

where

$$g(C_1, C_2, C_3, C_4) = C_1 + \frac{C_2}{C_3} + 4.25 \frac{C_2}{C_4}$$

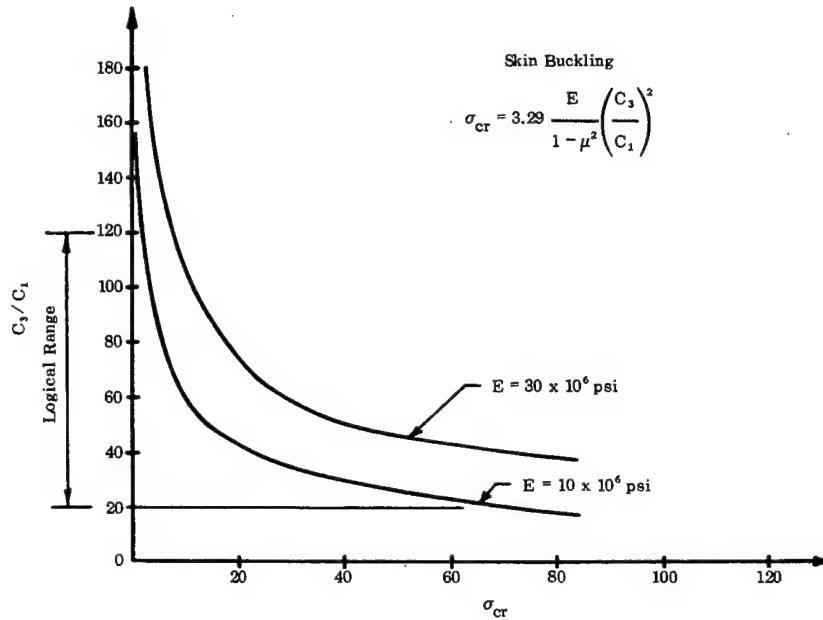
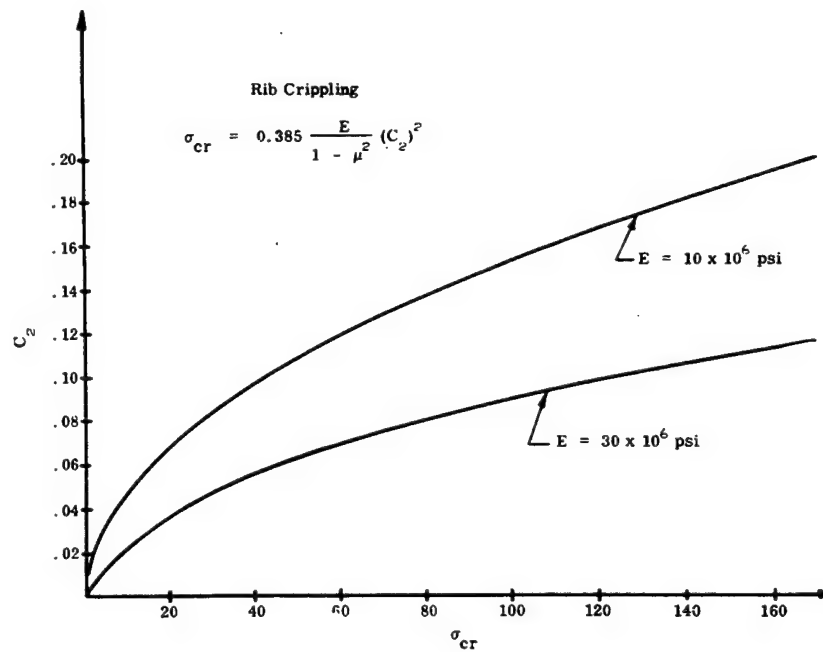
Substituting the value of  $b_w$  into the average thickness equation results in

$$t_{ave} = \frac{g(C_1, C_2, C_3, C_4)}{[f(C_1, C_2, C_3, C_4)]^{1/2}} \left( \frac{N_z R}{2CE} \right)^{1/2}$$

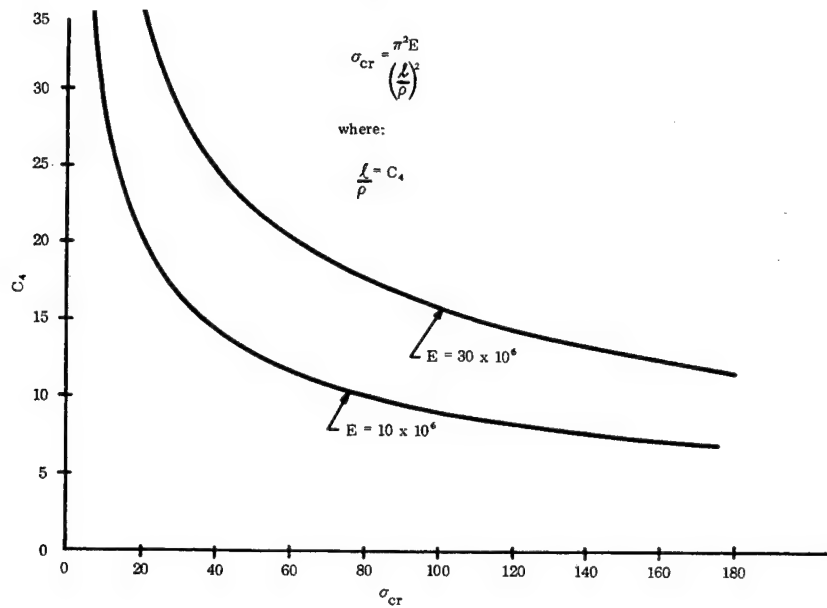
In order for the average thickness, and consequently the weight, to be a minimum, the following ratio must be maximum:

$$\frac{[f(C_1, C_2, C_3, C_4)]^{1/2}}{g(C_1, C_2, C_3, C_4)} \rightarrow \text{maximum}$$

The first step in determining a logical range of  $C_1$ ,  $C_2$ ,  $C_3$ , and  $C_4$  is to investigate skin buckling, which is dependent on the ratio  $C_3/C_1$ . A plot of critical skin buckling versus  $C_3/C_1$  was constructed and is shown in figure 72 (from ref. 64). Based on this plot, a range of  $C_3/C_1$  from 20 to 120 is sufficient to cover a wide range of allowable stress levels. Using  $C_1$  from 0.05 to 0.09 and  $C_3$  from 2 to 6 will result in the desired range of  $C_3/C_1$ . Similarly, a plot of  $C_2$ -versus-critical rib-crippling stress is constructed to determine the range of  $C_2$  to be

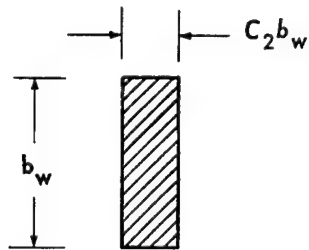
FIGURE 72.—Critical skin buckling versus  $C_3/C_1$ .FIGURE 73.— $C_2$  versus critical rib crippling.



FIGURE 74.—Critical Euler stress levels versus  $C_4$ .

investigated (fig. 73 from ref. 64). Based on this plot, a reasonable range of  $C_2$  is from 0.05 to 0.15.

Since  $C_4$  is a measure of ring spacing, panel buckling must be investigated to determine the range of values. Due to the complexity of the panel-buckling equation, however, this form of instability will be simplified by considering the stringers as Euler columns simply supported between rings. Values of the critical Euler stress levels versus  $C_4$  are plotted in figure 74 (from ref. 64). The value of  $l/\rho = C_4$  was arrived at as follows:



Stringer cross section

$$\rho^2 = \frac{I}{A} = \frac{C_2 b_w^4}{12 C_2 b_w^2}$$

$$\rho = \frac{1}{\sqrt{12}} b_w$$

$$l = C_4 b_w \text{ (ring spacing)}$$

Therefore,  $l/\rho = 3.42 C_4$ . Upon investigating the curve, it was concluded that the logical range of  $C_4$  was from 10 to 30.

**Development of Weight Equation**

In order to calculate the weight of the cylinder, the average smeared-out thickness, including the circumferential rings, is

$$t_{ave} = \left( C_1 + \frac{C_2}{C_3} + 4.25 \frac{C_2}{C_4} \right) b_w$$

The weight per surface area equals  $t_{ave}\rho F_b$ , where  $F_b$ , the fabrication factor accounting for noncalculated items, equals 1.20.

**SYMBOLS**

$N_x$	axial load per inch, lb/in.
$N_y$	hoop load per inch, lb/in.
$R$	radius of cylinder, in.
$L$	length of cylinder, in.
$b_w$	depth of rectangular stringers, in.
$t_s$	skin thickness, in.
$t_{ws}$	thickness of rectangular stringers, in.
$b_s$	spacing of rectangular stringers, in.
$b_r$	spacing of circumferential rings, in.
$t$	thickness of cylinder shell wall, in.
$d$	stringer spacing, in.
$l$	ring spacing, in.
$J_r$	torsional constant for ring, in. <sup>4</sup>
$J_s$	torsional constant for stringer, in. <sup>4</sup>
$G$	shear modulus, psi
$E$	modulus of elasticity, psi
$\mu$	Poisson's ratio
$A_s$	area of stringer, in. <sup>2</sup>
$A_r$	area of ring, in. <sup>2</sup>
$I_s$	moment of inertia of stringer, in. <sup>4</sup>
$I_r$	moment of inertia of ring, in. <sup>4</sup>
$\bar{Z}_r$	distance from centroid of ring to middle surface of shell, positive if stiffener lies on external surface of shell, in.
$\bar{Z}_s$	distance from centroid of stiffener to middle surface of shell, positive if ring lies on external surface of shell, in.
$\psi_s$	indicates whether stringers are external or internal to the skin surface; -1 if internal, +1 if external
$\psi_r$	indicates whether rings are external or internal to the skin surface; -1 if internal, +1 if external
$m$	number of half waves in cylinder buckle pattern in longitudinal direction
$n$	number of full waves in cylinder buckle pattern in circumferential direction

$C$	buckling correction factor
$A_{11}$	extensional stiffness in longitudinal direction, lb/in.
$A_{22}$	extensional stiffness in circumferential direction, lb/in.
$A_{33}$	shear stiffness, lb/in.
$D_{11}$	flexural stiffness in longitudinal direction, in.-lb
$D_{22}$	flexural stiffness in circumferential direction, in.-lb
$D_{33}$	torsional stiffness, in.-lb
$\sigma$	stress level, psi

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## Glossary

**Anisotropic**—Having properties that differ in several directions; possessing several natural axes, i.e., preferred directions with respect to a particular property or properties.

**Anticlastic**—Having opposite curvatures; having the center of principal radii at different sides of the observed tangent plane.

**Cementation (diffusion)**—Process for application of a coating metal by means of diffusion at elevated temperatures. The base metal is heated with a powdered coating metal to a temperature high enough to permit diffusion of fine particles.

**Cermet**—Composite having ceramic grains embedded in a metal matrix. Structural constituent accounts for as much as 30 percent of total value.

**Chemically rigidified structure**—Structure given rigidity through chemical reaction.

**Cladding**—Process by which a dense homogeneous layer of metal is bonded firmly and permanently to the base metal on one or two sides.

**Composite (composite material)**—Mixture or combination of two or more macro-constituents that differ in form and/or material composition and that are essentially insoluble in one another.

**Composite structures**—Material systems, designed and produced for a given application, that are the finished structures or products themselves.

**Criterion (merit) function**—Mathematical equation expressing all considerations associated with selecting the prime solution of a design problem; the function to be optimized.

**Design**—Process of evolving a configuration to perform specific functional requirements.

**Design process**—Sequence of operation by which the design configuration is evolved.

**Dispersion-hardened-alloy composite**—Composite having hard particles (usually of submicron size) of a structural constituent dispersed in a matrix of softer metal. Structural constituent is usually less than 3 percent by volume.

**Elastic recovery structures**—Structures utilizing the basic elastic properties of the materials to deploy and provide structural rigidity.

**Fiber (filament) composite**—Composite of fibers in continuous or discrete filament form embedded in a continuous matrix.

**Filament-wound structure**—Structure formed by draining continuous fiber (filament) through a resin bath and winding continuously onto a form, or mandrel, corresponding in shape to the inner structure of the fabricated part.

**Filled (skeletal) composite**—A continuous three-dimensional constituent having a random network of open pores or passages, cells, or an ordered honeycomb filled with another constituent.

**Flake composite**—Composition of flat particles or flakes, usually of an isotropic material, held together by an interface binder or embedded in a continuous matrix.



- Fully stressed design**—Structural members or elements are proportioned by equalizing the stress in any member (or element) in at least one loading condition.
- Functional constraint**—Mathematical description of the archetype of the proposed design.
- Geodesic dome**—Type of roof employing the triangle and pentagon used in subdivision with bars of equal length. Vertexes of the curved figures forming the structures mark the arcs of great circles (geodesics).
- Inflatable structure**—Fabric or film envelope maintaining structural integrity by internal pressurization.
- Interface**—Surface forming the common boundary of the constituents.
- Isostatics**—Plot of principal stress lines.
- Isotensoid**—Filamentary structure (pressure vessel) in which the filaments are oriented so that there is equal and uniform tension in each fiber.
- Isotropic**—Having no preferred direction with respect to a particular property or properties; having no natural axes.
- Laminar composite**—Composition of layers of single constituents bonded as superimposed layers.
- Lattice dome**—Shell system constructed from thin laths in which a flat square grid is deformed into a spatially curved surface.
- Matrix**—Body constituent of a composite; the “enclosure” material.
- Membranes**—Very thin stress skins capable of resisting only tension.
- Micromechanics**—Analysis of composite behavior based on the individual constituents and their interactions.
- Minimum-weight design**—Arrangement of structural elements in which all the design requirements such as stresses, deflections, and geometric constraints are satisfied while the total weight of the entire structure is minimized.
- Monolithic material**—A simple material.
- Optimization**—Technique for finding the best combination of design parameter values that satisfy the design requirements.
- Orthotropic material**—Material possessing three natural axes at right angles.
- Particulate composite**—Composition of minute particles, usually of uniform shape, embedded in a continuous matrix.
- Regional constraint**—Sets the allowable limits on design parameters or derived groups of parameters that represent the more complex attributes of the proposed object.
- Roving**—Untwisted grouping of filaments.
- Sandwich**—Construction comprising a combination of alternating dissimilar, simple or composite materials, assembled and intimately fixed in relation to each other so as to use the properties of each to the specific structural advantage of the whole assembly.
- SAP**—Sintered aluminum powder.
- Self-lubricating alloy**—Dispersion of dry lubricant powder in a metal matrix.
- Semimonocoque**—Skin structure stiffened by a number of reinforcing elements.
- Structural constituent**—Constituent determining the internal structure of the composite.
- Synthesis**—Fitting together design elements or separate concepts to produce an integrated whole.
- Unfurlable structure**—Body that is mechanically deployed by hinges, sliding sections, and telescoping members.
- Viscoelastic material**—Material having behavior characteristic of fluids while maintaining some of the rigidity of solids.

## Symbols

### Notations

$A$	area, in. <sup>2</sup> ; subspace of acceptable design points
$b$	spacing; width, in.
$C$	coefficient; buckling correction factor; optimum design parameter
$c$	specific heat, Btu/lb/°F
$D$	structural design index, psi
$d$	stringer spacing, in.; diameter, in. or ft
$E$	modulus of elasticity, psi
$e$	emissivity
$F$	force, lb; merit function to be optimized
$G$	shear modulus, psi; combined constraint boundary between $A$ and $U$
$g_i$	$i$ th constraint function
$h$	depth, in.
$I$	moment of inertia, in. <sup>4</sup>
$J$	torsional constant, in. <sup>4</sup>
$K$	coefficient; shape factor
$k$	thermal conductivity, Btu/hr/ft <sup>2</sup> /°F; coefficient
$k/c_p$	diffusivity, ft <sup>2</sup> /hr
$L$	length, in. or ft
$l$	spacing, in. or ft
$M$	material efficiency factor, pci/psi; moment, in.-lb or ft-lb
$m$	number of half waves in cylinder buckle pattern in longitudinal direction
$N$	edge or end loading, lb/in.
$n$	number of variables; number of full waves in cylinder buckle pattern in circumferential direction
$P$	load, lb; probability symbol
$p$	pressure, psi
$R$	radius, in. or ft
$S$	structural strength, psi; structural efficiency factor, nondimensional; design space
$t$	thickness, in.
$U$	criterion (merit) function; space of unacceptable design points

$V$	volume, in. <sup>3</sup> or ft <sup>3</sup>
$W$	weight, lb; weight efficiency factor, pci/psi
$w$	width, in. or ft
$X$	variable; design parameter
$x$	coordinate
$y$	coordinate
$\alpha$	coefficient of thermal expansion, in./in.-°F
$\eta$	plasticity reduction factor, nondimensional
$\mu$	Poisson's ratio, nondimensional
$\xi$	variable; coordinates
$\rho$	density, pci
$\sigma$	stress, psi
$\sigma_{cy}$	compressive yield strength, psi
$\phi$	regional constraint
$\psi$	functional constraint

*Subscripts*

$c$	core
$f$	face sheet
$r$	rib
$s$	stringer
$w$	depth
$x$	$x$ -direction
$y$	$y$ -direction

*Special Notations*

$A_{11}$	Extensional stiffness in longitudinal direction, lb/in.
$A_{22}$	Extensional stiffness in circumferential direction, lb/in.
$A_{33}$	Shear stiffness, lb/in.
$D_{11}$	Flexural stiffness in longitudinal direction, in.-lb
$D_{22}$	Flexural stiffness in circumferential direction, in.-lb
$D_{33}$	Torsional stiffness, in.-lb
$\bar{Z}_r$	Distance from centroid of ring to middle surface of shell; positive if ring lies on external surface of shell, in.
$\bar{Z}_s$	Distance from centroid of stringer to middle surface of shell; positive if stiffener lies on external surface of shell, in.
$\psi_r$	Indicates whether rings are external or internal to the skin surface; -1 if internal, +1 if external
$\psi_s$	Indicates whether stringers are external or internal to the skin surface; -1 if internal, +1 if external
$\epsilon$	Reads "contained in"

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